Journal of Statistical Physics, Vol. 49, Nos. 3/4, 1987

High-Temperature Expansions to Fifteenth Order

Ralph Z. Roskies[1] **and Penny D. Sackett**[1]

Received January 13, 1987

High-temperature series expansions of the susceptibility and second moment to 15th order are calculated for zero external field on the linear chain (LC), plane square (PSQ), simple cubic (SC), and body-centered cubic (BCC) lattices. Checks for specific models against pertinent work in the literature are detailed.

KEY WORDS: High-temperature series; high-temperature expansions; strong coupling series; strong coupling expansions.

1. INTRODUCTION

High-temperature series expansions have proven to be a powerful method of studying lattice models. In particular, much attention has been devoted to the comparison of predictions for critical exponents derived from series expansions and from renormalization group methods (see, e.g., Refs. 1).

The unrenormalized lattice strong coupling expansion in quantum field theory is directly related to the high-temperature series expansion of statistical mechanics, with the square of the bare coupling playing the role of temperature. The expansions are lattice- and potential-dependent, of course, but the dependence on potential can be absorbed into vertex functions, which then can be left general, so that the remaining coefficients are only a function of lattice geometry.

In this paper, we use a diagrammatic prescription very similar to that of Bender *et al.*[(2)] to obtain the high-temperature (strong coupling) expansion of any scalar spin theory (quantum field theory) with nearest neighbor interactions with an even spin density (potential). We compute the susceptibility χ and the second moment μ_2 in zero external field to 15th order in the expansion parameter for four diffeent lattices: the linear chain (LC), the

[1] Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260.

0022-4715/87/1100-0447$05.00/0

plane square lattice (PSQ), the simple cubic lattice (SC), and the body-centered cubic lattice (BCC).

The power of this method is derived from three characteristics of the approach:

1. The graph counting and combinatorial weighting have been completely automated, so that computer resources (and not human stamina and fallibility) limit the order of perturbative calculations. The graph counting method is new.
2. The absence of tadpoles (lines beginning and ending at the same vertex) greatly reduces the number of graphs that must be considered to a given order.
3. The spin-density-dependent (potential-dependent) information is entirely contained in the vertex functions, so that once the pertinent graphs have been enumerated, the series is applicable to any spin-density (potential) that is even.

In Section 2 we give a brief description of the expansion and how it may be reexpressed as a series of terms each of which factors into separate lattice-dependent and spin-density-dependent terms. In Section 3 we outline how one-vertex irreducible subgraphs (OVIS) may be used to generate the set of the one-particle irreducible graphs that are then assembled to form all graphs that contribute to the 15th-order expansion. The algebraic procedure used to form the susceptibility and second moment from the one-particle irreducible graphs is described in Section 4. The results, which extend the tenth-order tables of Kincaid *et al.*,[3] appear in a similar tabular form in Section 5. A discussion of the consistency checks made on them can be found in Section 6. The Appendix describes in some detail the new algorithm used to generate the OVIS.

The results of this paper have already been used to study hyperscaling in the Ising model.[4]

2. THE DIAGRAMMATIC EXPANSION

Using the language of quantum field theory, we begin with a generating functional (partition function) on a d-dimensional Euclidean lattice

$$Z[J] = \int [D\phi] \exp \sum_i a^d \left(\frac{\phi_i(\phi_{i+1} + \phi_{i-1} - 2d\phi_i)}{2a^2} - V(\phi_i) - J_i\phi_i \right) \tag{2.1}$$

which is the lattice version of the continuous generating functional

$$Z[J] = \int [D\phi] \exp \left\{ -\int d^4x \left[\frac{1}{2} (\partial\phi)^2 + V(\phi) + J\phi \right] \right\} \tag{2.2}$$

The potential $V(\phi_i)$ is a local one, which depends only on the value of the field at site i; $J_i\phi_i$ is a source term. The "off-diagonal" piece of the kinetic energy operator may be extracted from the action to act as an operator on the remaining portion of the functional integral,

$$Z[J] = \exp \sum_i \frac{v}{2} \left[\frac{\delta}{\delta J_i} \left(\frac{\delta}{\delta J_{i+1}} + \frac{\delta}{\delta J_{i-1}} \right) \right] \times \int [D\phi] \exp\left\{ - \sum_i a^d \left[V(\phi_i) + \frac{d\phi_i^2}{a^2} + J_i\phi_i \right] \right\} \tag{2.3}$$

with $v = a^{-(d+2)}$. This produces a formal expansion for the generating functional, from which the n-point Green's functions of the theory (correlation functions) can be obtained by a simple diagrammatic procedure.

In a strong coupling expansion, the diagrams are organized by increasing powers of v, the expansion parameter. This parameter counts the number of free inverse propagators (lines) in the graph, in contrast to the organization of graphs by increasing powers of g (vertices) in weak coupling expansions. The n-point connected Green's functions $G_n(\phi_1, \phi_2, ..., \phi_n)$ of the theory at zero source field can then be computed from $Z(J)$ by taking functional derivatives with respect to the external source $J(\phi)$:

$$G_n(\phi_1, \phi_2, ..., \phi_n) = \frac{\delta}{\delta J(\phi_1)} \cdots \frac{\delta}{\delta J(\phi_n)} \ln Z[J] \bigg|_{J=0} \tag{2.4}$$

Normalization of the generating functional is of no concern, therefore, if we restrict our attention to Green's functions.

Since we consider only potentials that are even in the field variable ϕ, all vertices represent the intersection of an even number of bare propagators. As in the usual high-temperature series expansion, we have only factored out the off-diagonal portion of the kinetic energy operator, and as a result there are no graphs containing tadpoles.

The n-point connected Greens' functions can be constructed from the formal expansion through a given order in v by the use of a set of diagrammatic rules. Following Bender *et al.*,[2] we list these rules for constructing n-point Green's functions to order N in the internal line counter v:

1. Draw all connected graphs with no tadpoles having a total of n external lines and N internal lines. The vertices of these graphs must be the intersection of an even number of lines (external or internal). This enumeration of graphs is general to all even scalar theories in zero external field.

2. Associate with every $2p$ vertex the amplitude λ_{2p} given by

$$\lambda_{2p} = v^p W_{2p}$$

where the W_{2p} are the vertex functions defined by

$$\exp \sum_{p=0}^{\infty} \frac{J^{2p} W_{2p}}{(2p)!} = \frac{\int d\phi \exp\{-a^d[V(\phi) + d\phi^2/a^2 + J\phi]\}}{\int d\phi \exp\{-a^d[V(\phi) + d\phi^2/a^2]\}} \tag{2.5}$$

The vertex functions carry all potential-dependent information, but are lattice-independent.

3. With each graph, associate two different coefficients: the topological symmetry number and the free multiplicity.

(a) The topological symmetry number is the inverse of the number of ways that an identical graph may be obtained by relabeling the internal lines and vertices of the graph. This number is potential- and lattice-independent.

(b) The free multiplicity counts the number of ways that a given graph with fixed external lines may be embedded onto the chosen lattice with the vertices corresponding to lattice sites and the internal lines connecting neighboring sites. This number is potential-independent but lattice-dependent and may be zero, in fact, for some graphs on some lattices.

4. Form the product of the vertex amplitude, the topological symmetry number, the free multiplicity, and $v^{-n/2}$ to arrive at the total contribution to the series from each graph.

5. Sum the contributions of all graphs.

We implemented these rules through 15th order for several different lattice geometries on a DEC-10 using primarily FORTRAN, and to a lesser extent the symbolic manipulation routines ASHMEDAI[5] and REDUCE.[6] Before describing our technique for building all connected graphs from a smaller set of graphs, let us illustrate the rules above with an example.

Suppose we wish to compute the susceptibility [that is, the two-point correlation function $G_2(\mathbf{x}, \mathbf{y})$ when summed over all external points $(\mathbf{y})$] for an even scalar theory on a plane square lattice through third order in our perturbation parameter v. We begin by enumerating all graphs relevant to this order, i.e., those containing no more than three internal lines. They are shown in Fig. 1.

The topological symmetry number and free multiplicity are also shown for each of these graphs. Notice that the topological symmetry number depends only on the shape of the graph, but the free multiplicity depends

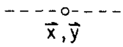
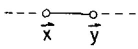
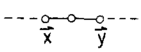
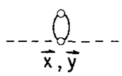
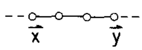
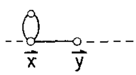
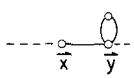
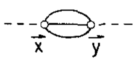
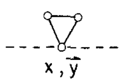
ORDER	GRAPH	TOPOLOGICAL SYMMETRY NUMBER	FREE MULTIPLICITY [plane square lattice]	$\lvert\vec{x}-\vec{y}\rvert$	CONTRIBUTION TO $\chi \equiv \sum_{\vec{y}} G_2(\vec{x},\vec{y})$
0		1	1	0	W_2
1		1	1	1	$4vW_2^2$
2		1	4 2 1	0 $\sqrt{2}$ 2	$16v^2W_2^3$
2		$1/2$	4	0	$2v^2W_2W_4$
3		1	9 3 1	1 $\sqrt{5}$ 3	$64v^3W_2^4$
3		$1/2$	4	1	$8v^3W_2^2W_4$
3		$1/2$	4	1	$8v^3W_2^2W_4$
3		$1/6$	1	1	$2/3\, v^3W_4^2$
3		$1/2$	0	0	0

Fig. 1. All graphs contributing through third order to the strong coupling expansion of the two-point Green's function $G_2(\mathbf{x}, \mathbf{y})$. External lines are dashed; internal lines are solid. The topological symmetry number of each graph is shown with the free multiplicity of the graph for different embeddings $(\mathbf{x}, \mathbf{y})$ on a plane square lattice. The total contribution of each graph to the susceptibility through third order is also given.

on the shape of the graph, the type of lattice, and the relative location of the external legs. For example, the graph with two internal lines and three vertices can be placed on a plane square lattice in only one way if the external points are (0, 0) and (2, 0). But the same graph can be laid onto the plane square lattice in two different ways if the external points are (0, 0) and (1, 1). Note, too, that the last graph in the list does not contribute on a plane square lattice, but would contribute on a plane triangular lattice.

Counting the number of internal lines to arrive at the appropriate power of v is trivial, and the sum over one of the external points $\mathbf{y}$ (up to three lattice units away from $\mathbf{x}$) of all contributions then gives the susceptibility to third order.

3. ONE-VERTEX IRREDUCIBLE SUBGRAPHS

In order to automate the graph bookkeeping to the largest possible extent, we first build a subset of graphs, called the kernel. These graphs are constructed according to the rules of the previous section for the two-point function, with the added restriction that they be one-particle irreducible. One-particle irreducible graphs are those that cannot be separated into two disjoint graphs by the removal of a single internal line. From the kernel $K(\mathbf{x}, \mathbf{y})$, other physical quantities of interest, such as the full two-point function $G_2(\mathbf{x}, \mathbf{y})$ may be built. The kernel itself is built from smaller building blocks, which we call one-vertex irreducible subgraphs (OVIS).[7] The set of OVIS is the set of all one-particle irreducible graphs in the two-point function for which the removal of any internal vertex will not separate the graph into two disjoint pieces only one of which contains both external legs. Such a set of graphs is, in fact, equivalent to the full kernel if the vertex functions are appropriately redefined. In this section we describe the algorithm for generating OVIS and the vertex redefinition.

All OVIS through sixth order in the internal line counter are shown in Fig. 2, with the redefined vertices indicated as dots centered in open circles. For each graph, the topological symmetry number is shown along with the free multiplicity for all possible embeddings on a plane square lattice. The degrees of a graph are the number of lines that enter each of its vertices. The sum of the degrees must equal $2N+n$, where N is the order of the graph and n is the number of external legs. In the Appendix we describe our new algorithm for generating all such OVIS to a given order together with their topological symmetry numbers and lattice-dependent free multiplicities.

Having generated the OVIS, the task of applying the appropriate vertex redefinition remains. The redefinition must build the full kernel by the addition of graphs that are one-vertex reducible. The redefined vertices, the M_{2p}, can then be expanded as a power series in the previously defined W_{2p}. This redefinition can be thought of pictorally as beginning with a "bare" W_{2p} vertex and attaching vacuum "decorations" that have an even number of internal lines and bare W_{2p} internal vertices. The sum of all possible decorations to a given order form the "decorated" M_{2p} vertex functions of the OVIS. Thus, the set of OVIS hide in their decorated vertices the elements of the kernel that are one-vertex reducible.

Shown in Fig. 3 is the pictorial expansion through fourth order of the

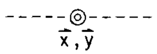
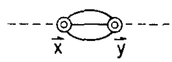
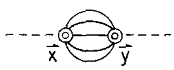
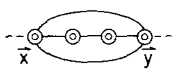
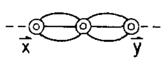
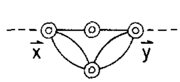
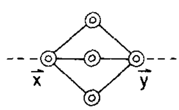
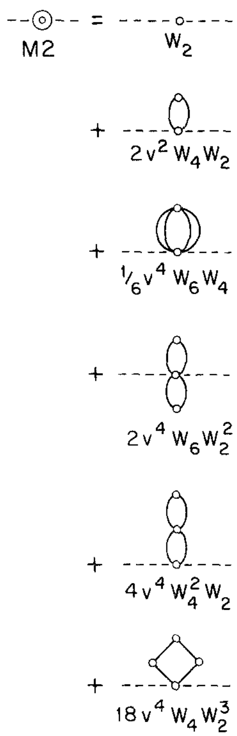
ORDER	GRAPH	TOPOLOGICAL SYMMETRY NUMBER	FREE MULTIPLICITY [plane square lattice]	$\lvert\vec{x}-\vec{y}\rvert$	DEGREES
0		1	1	0	2
3		1/6	1	1	4,4
5		1/120	1	1	6,6
5		1/2	9	1	4,4,2,2
6		1/36	4 2 1	0 $\sqrt{2}$ 2	6,4,4
6		1/4	16 4 1	0 $\sqrt{2}$ 2	4,4,4,2
6		1/6	64 8 1	0 $\sqrt{2}$ 2	4,4,2,2,2

Fig. 2. All one-vertex irreducible subgraphs contributing through sixth order to the strong coupling expansion. External lines are dashed; internal lines are solid. The redefined, or "decorated," vertices M_{2p} are shown as dots centered in open circles. The topological symmetry number of each graph is shown with the free multiplicity for different embeddings on a plane square lattice. The degrees are also shown for each graph.

decorated vertex of lowest degree, M_2, into its bare vertex decorations. The algebraic expansion is also given to the same order for comparison. Each decoration is of even order and is associated with its own topological symmetry number. The bare vertices of the decorations are all internal and their positions are therefore summed over freely to obtain the free multiplicity sum, which, although independent of the external positions **x** and **y**, is, of course, lattice-dependent. Notice that for decorations the sum of the degrees is again equal to $2N+n$, where N is now the number of internal lines added by the decoration and n is the degree of the decorated vertex.

We choose this redefinition for the vertex functions of the OVIS since a simple substitution of the series expansions in the bare W_{2p} for the decorated M_{2p} produces composite graphs for which the topological



DECORATION	ORDER	TOPOLOGICAL SYMMETRY NUMBER	FREE MULTIPLICITY [plane square lattice]	DEGREES
M2 = W_2	0	1	1	2
+ $2v^2 W_4 W_2$	2	1/2	4	4,2
+ $\frac{1}{6} v^4 W_6 W_4$	4	1/24	4	4,4
+ $2v^4 W_6 W_2^2$	4	1/8	16	6,2,2
+ $4v^4 W_4^2 W_2$	4	1/4	16	4,4,2
+ $18v^4 W_4 W_2^3$	4	1/2	36	4,2,2,2

Fig. 3. All decorations contributing through fourth order to the decorated vertex M_2. Lines associated with the decorated vertex are dashed; lines internal to the decoration are solid. The decorated vertex is shown as a dot in an open circle; the bare vertices are shown as dots. The order of each decoration is shown with its topological symmetry number. The free multiplicity sum for each decoration is shown here for embeddings on a plane square lattice. The degrees for each decoration are also indicated.

symmetry numbers and free multiplicities are a simple product of those for the constituent OVIS and decorations. This procedure again lends itself to automation.

4. THE SUSCEPTIBILITY χ AND THE SECOND MOMENT μ_2

By expanding the decorated vertices of the set of OVIS to a given order, one generates the full one-particle irreducible kernel $K(\mathbf{x}, \mathbf{y})$. From this kernel, the following then may be easily found: the two-point function

$G_2(\mathbf{x}, \mathbf{y})$; its sum over external $\mathbf{y}$, namely the susceptibility $\chi = \sum_{\mathbf{y}} G_2(0, \mathbf{y})$; and the second moment of the two-point function $\mu_2 = \sum_{\mathbf{y}} G_2(0, \mathbf{y})\, \mathbf{y}^2$.

The two-point function $G_2(\mathbf{x}, \mathbf{y})$ is the sum of all graphs with vertices connected to external lines at $(\mathbf{x}, \mathbf{y})$. All such graphs can be formed from the kernel $K(\mathbf{x}, \mathbf{y})$, and the bare propagator $P(\mathbf{x}, \mathbf{y})$ by forming the following series, which can then be truncated at the desired order:

$$G_2(\mathbf{x}, \mathbf{y}) = K(\mathbf{x}, \mathbf{y}) + \sum_{\mathbf{x}', \mathbf{x}''} K(\mathbf{x}, \mathbf{x}')\, P(\mathbf{x}', \mathbf{x}'')\, K(\mathbf{x}'', \mathbf{y}) + \cdots \qquad (4.1)$$

The bare propagator is simply related to the line counter v, since

$$P(\mathbf{x}, \mathbf{y}) = v\{\delta_{\mathbf{x}, \mathbf{y}+\hat{\mathbf{u}}} + \delta_{\mathbf{x}, \mathbf{y}-\hat{\mathbf{u}}}\} \qquad (4.2)$$

where $\hat{\mathbf{u}}$ is a vector that lies along one of the positive lattice axes and connects neighboring lattice points.

The susceptibility χ is the sum over one external point of the two-point function

$$\chi \equiv \sum_{\mathbf{y}} G_2(0, \mathbf{y}) \qquad (4.3)$$

If we further define an integrated kernel K_0 as

$$K_0 \equiv \sum_{\mathbf{y}} K(0, \mathbf{y}) \qquad (4.4)$$

then, using the translation invariance of the lattice, we may write the susceptibility as the following power series in K_0:

$$\chi = \sum_{p=0}^{\infty} (2\eta v)^p\, K_0^{p+1} \qquad (4.5)$$

where η is the number of axes of symmetry for the lattice. Since K_0 itself is a power series in v, we must remember to keep in each term of Eq. (4.5) only those pieces that contribute to the desired order of expansion.

As an example, consider the susceptibility through third order in v on a plane, square lattice ($\eta = 2$). The one-particle irreducible graphs contributing to the kernel at this order are shown in Fig. 4. The integrated kernel K_0 is thus given by

$$K_0 = W_2 + 2v^2 W_2 W_4 + \tfrac{2}{3} v^3 W_4^2 \qquad (4.6)$$

The reader may verify (against Fig. 1) that substitution of this expression into Eq. (4.5), keeping terms no higher that v^3, will reproduce the full set of graphs with their corresponding contribution to the susceptibility.

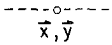
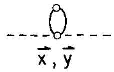
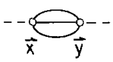
ORDER	GRAPH	TOPOLOGICAL SYMMETRY NUMBER	FREE MULTIPLICITY [plane square lattice]	$\lvert\vec{x}-\vec{y}\rvert$	CONTRIBUTION TO $K_0 \equiv \sum_{\vec{y}} K(0,\vec{y})$
0		1	1	0	W_2
2		$1/2$	4	0	$2v^2 W_2 W_4$
3		$1/6$	1	1	$2/3\, v^3 W_4^2$

Fig. 4. All kernel graphs through third order on a plane square lattice. External lines are dashed; internal lines are solid. The contribution of each graph to the integrated kernel K_0 is shown.

The second moment μ_2 is defined as

$$\mu_2 \equiv \sum_{\mathbf{y}} G_2(0, \mathbf{y})\, \mathbf{y}^2 \tag{4.7}$$

and is not translationally invariant. It is related to the Fourier transform $\tilde{G}_2$ of G_2 by

$$\mu_2 = -\nabla_p^2 \tilde{G}_2(\mathbf{p})\,|_{\mathbf{p}=0} \tag{4.8}$$

From Eq. (4.1) we have

$$\tilde{G}_2(\mathbf{p}) = \tilde{K}(\mathbf{p}) + \tilde{K}(\mathbf{p})\, \tilde{P}(\mathbf{p})\, \tilde{K}(\mathbf{p}) + \cdots \tag{4.9}$$

or

$$\tilde{G}_2(\mathbf{p}) = \frac{\tilde{K}(\mathbf{p})}{1 - \tilde{P}(\mathbf{p})\, \tilde{K}(\mathbf{p})} \tag{4.10}$$

Now using the fact that the two-point function is an even function of **p**, (lattice isotropy), we apply $-\nabla_p^2$ to Eq. (4.10) to arrive at

$$\mu_2 = \left.\frac{-[\nabla_p^2 \tilde{K}(\mathbf{p})]}{1 - \tilde{P}(\mathbf{p})\, \tilde{K}(\mathbf{p})}\right|_{\mathbf{p}=0} - \tilde{K}(\mathbf{p}) \left.\left\{ \frac{\tilde{P}(\mathbf{p})[\nabla_p^2 \tilde{K}(\mathbf{p})] + [\nabla_p^2 \tilde{P}(\mathbf{p})]\, \tilde{K}(\mathbf{p})}{[1 - \tilde{P}(\mathbf{p})\, \tilde{K}(\mathbf{p})]^2} \right\}\right|_{\mathbf{p}=0} \tag{4.11}$$

Note that

$$\tilde{P}(0) = \sum_{\mathbf{y}} P(0, \mathbf{y}) = 2v\eta$$
$$\tilde{K}(0) = \sum_{\mathbf{y}} K(0, \mathbf{y}) \equiv K_0 \tag{4.12}$$
$$-\nabla_p^2 \tilde{P}(0) = \sum_{\mathbf{y}} P(0, \mathbf{y})\, y^2 = 2v\eta$$

and further, define K_2 to be the second moment of the kernel, that is

$$-\nabla_p^2 \tilde{K}(\mathbf{p})\,|_{p=0} = \sum_{\mathbf{y}} K(0, \mathbf{y})\, \mathbf{y}^2 \equiv K_2 \tag{4.13}$$

So, as a power series in K_0,

$$\mu_2 = \frac{K_2 + 2v\eta K_0^2}{(1 - 2v\eta K_0)^2} = (K_2 + 2v\eta K_0^2) \sum_{p=0}^{\infty} (p+1)(2v\eta K_0)^p \tag{4.14}$$

Since K_2 begins at order v, so does the second moment μ_2.

As an exercise, the reader may verify that through third order on a plane square lattice the second moment is

$$\mu_2 = 4vW_2^2 + 32v^2 W_2^3 + v^3(192W_2^4 + 16W_2^2 W_4 + \tfrac{2}{3}W_4^2) \tag{4.15}$$

Using ASHMEDAI and REDUCE, the series in Eqs. (4.5) and (4.14) were expanded algebraically and truncated at the 15th order for four different lattice geometries. The results are presented in tabular form at the end of the paper.

5. THE RESULTS

The results of our strong coupling expansion of the susceptibility χ and second moment μ_2 are given in Tables I and II respectively. We have tabulated the series, order by order, in a form that is completely analogous to the tenth-order tables of Kincaid *et al.*,[3] whose work we extend here through 15th order. Note that Kincaid *et al.* computed χ and μ_2 on the more complicated triangular and face-centered cubic lattices, and also computed $\partial^2\chi/\partial H^2$ for all lattice geometries considered; we did not.

The series may be reconstructed from the tables by multiplying each partition by the corresponding coefficient and summing through the desired order. The coefficients contain all the lattice-dependent information and are tabulated for the four lattice geometries of this work: linear chain

(LC), plane square (PSQ), simple cubic (SC), and body-centered cubic (BCC).

The partitions are grouped by leading power of the expansion parameter v and indicate (in ascending order) the degree of each strong coupling moment I_{2n} found in that term. The moments are related to the vertex functions W_{2n} through

$$\sum_{n=0}^{\infty} \frac{J^{2n} I_{2n}}{(2n)!} = \exp \sum_{n'=0}^{\infty} \frac{J^{2n'} W_{2n'}}{(2n')!} \tag{5.1}$$

and contain, therefore, all the potential-dependent information. By differentiating both sides of Eq. (5.1) with respect to J, we obtain

$$\sum_{n=1}^{\infty} \frac{I_{2n} J^{2n-1}}{(2n-1)!} = \left(\sum_{n'=1}^{\infty} \frac{W_{2n'} J^{2n'-1}}{(2n'-1)!} \right) \left(\sum_{m=0}^{\infty} \frac{I_{2m} J^{2m}}{(2m)!} \right) \tag{5.2}$$

Now, by collecting like terms in powers of J, we arrive at the following recursive formula for the expansion of the I_{2n} as a power series in the W_{2n}:

$$I_{2n} = (2n-1)! \left\{ \sum_{n'=1}^{n} \frac{W_{2n'}}{(2n'-1)!} \frac{I_{2(n-n')}}{[2(n-n')]!} \right\} \tag{5.3}$$

Note that for convenience, and to ensure that all coefficients listed in the tables are integers, we have multiplied each coefficient in the leading nth order by $n!$.

To illustrate the format via an example, consider the susceptibility on a plane square lattice to third order. Table I indicates the following series in terms of the moments I_{2n}:

$$\chi = I_2 + 4vI_2^2 + \frac{v^2}{2!}(20I_2^3 + 4I_2I_4) + \frac{v^3}{3!}(132I_2^4 + 72I_2^2I_4 + 4I_4^2) \tag{5.4}$$

Using Eq. (5.3), we find

$$I_2 = W_2, \qquad I_4 = W_4 + 3W_2^2 \tag{5.5}$$

The reader may verify that substitution yields the expression for the susceptibility on a plane square lattice through third order previously found in Fig. 1.

The complete set of tables is available from the authors in computer-ready form, via electronic mail or magnetic tape.

6. VERIFICATION

Our series extend an existing body of literature in strong coupling expansions in field theory and high-temperature expansions in statistical mechanics. For a description and comparison of the methods of expansion used in these two fields, see the Appendix of Ref. 7. We have checked our series against those in the literature wherever comparison is possible, and review the most stringent checks below. In all cases, our series agreed with those found elsewhere.

The most general work preceding ours is that of Kincaid *et al.*[3] Their series for the susceptibility χ and second moment μ_2 are identical to ours through tenth order (the highest computed by them) for all four of the lattice geometries we considered.

To check our results at an order higher than ten, we must rely on special choices of the potential. The spin-S Ising model has been well studied. The moments I_{2n} in this model can be easily derived, since there is no self-interaction term $[V(\phi)=0]$, and the off-diagonal nearest neighbor spin–spin interaction plays the role of the kinetic energy operator in lattice field theory. The internal line counter and expansion parameter v is replaced with the coupling parameter J/kTS^2, where J is now the spin exchange constant and the external field is the magnetic field H. With these identifications and Eqs. (5.1) and (2.5), the moments I_{2n} in the spin-S Ising model can be found from

$$\sum_{n=0}^{\infty} \frac{I_{2n} h^{2n}}{(2n)!} = \frac{\sinh[Y(h/2)]}{Y \sinh(h/2)} \tag{6.1}$$

(where $Y=2S+1$ and $h=H/kT$), by expanding the left-hand side and matching like powers of h.

For example, for the spin-1/2 Ising model, $Y=2$ and one finds from Eq. (6.1) that

$$I_{2n} = 1/(2)^{2n}, \qquad v = 4J/kT \tag{6.2}$$

With these substitutions, the linear chain (LC) spin-1/2 results are trivial and easily checked against closed-form expressions. Our plane square (PSQ) series for χ and μ_2 agree with those of Wu *et al.*[8] through 15th order. Our series on a simple cubic (SC) lattice match those of Sykes *et al.*[9] for χ through 15th order, those of Moore *et al.*[10] for μ_2 through order 12, and, of course, those of Roskies[4] for μ_2 through 15 orders. The most extensive spin-1/2 series for χ and μ_2 on a body-centered cubic (BCC) lattice are the 21st-order results of Nickel,[11] which agree through order 15 with our own.

In conclusion, we have extended through 15th order the strong coupling (high-temperature) series for the susceptibility χ and second moment μ_2 for models that are even in a scalar potential on four different lattice geometries: linear chain (LC), plane square (PSQ), simple cubic (SC), and body-centered cubic (BCC). Our algorithm for generating these series has the advantage of separating lattice-dependent from model-dependent considerations, and is computationally efficient. The graph enumeration scheme we use is new, and is described in the Appendix. Less general series in the literature are consistent with our own.

APPENDIX. GENERATION OF GRAPHS

The basic problem in a graphical enumeration scheme is to ensure that each different graph is counted once and only once. If one has an algorithm for generating graphs, one must generally develop a canonical form for graphs and accept a new graph only if it is in canonical form. One has also to assure that every canonical form is generated once and only once. This is a difficult problem (see, e.g., Ref. 12). It sometimes requires searching through a large list of already generated graphs to see that the newly generated one is different from all of those in the list.

But in our case, as explained in the text, we multiply the contribution of each graph by its topological symmetry number. So we can afford to generate a graph more than once, provided that we associate with it a pseudosymmetry number, which has the property that the sum of the pseudo-symmetry numbers for the graphs equivalent to a given graph is the correct symmetry number for that graph. This is usually wasteful, but we were able to use it to find a graph-generating algorithm that eliminated the need for keeping track of the graphs already encountered.

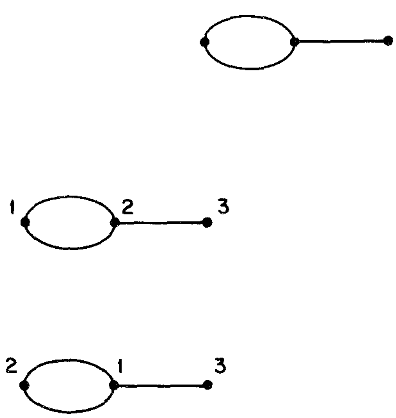
It is easy to find a canonical representation for a graph. Represent it by a symmetric connection matrix whose rows and columns represent the vertices and whose (i, j)th entry indicates the number of lines joining vertex i to vertex j. We can introduce an ordering of matrices (say lexical ordering reading row by row). A graph will be in canonical form if its corresponding matrix is the highest of all equivalent graphs obtained by an arbitrary permutation of its vertices. The problem then is to check whether a given matrix is lexically higher than all equivalent matrices obtained by an arbitrary permutation of its vertices. This is potentially very time-consuming, because the number of permutations grows rapidly with the size of the graph. There are trivial simplifications, such as ordering the vertices by their degree and considering only subgroups of permutations that permute only vertices of the same degree. This is still too time-consuming to be practical.

We present a graph-generating scheme that will sometimes generate copies of equivalent graphs, but will associate to them a pseudo-symmetry number that has the desired property mentioned above. As the degree of the graphs grows, the number of copies of equivalent graphs grows, too, but to the order we work (15 lines), this overcounting is not serious. It would be more severe if we went to much higher order (as Nickel[11] has done).

Suppose that a particular $N \times N$ connection matrix M is given. We label its vertices 1 through N, corresponding to its rows and columns. A matrix obtained by permuting the rows and column of M clearly represents the same graph, but may be represented by a different matrix. For example, the graph shown in Fig. 5 with two different vertex labelings can be represented by two different corresponding matrices.

What follows is the algorithm for accepting a particular connection matrix and computing its pseudo-symmetry number ps. First set ps = 1. Then examine the vertices in order, making sure that they have been chosen to satisfy certain criteria. At any stage we assume that NV vertices have already been chosen, and that they are numbered 1 to NV. The next vertex must satisfy the following conditions:

1. It must be the highest degree vertex remaining. If there are several, it must be the one connected to the lowest-numbered chosen vertex. If there are several, it must be the one also connected to the next-lowest-numbered chosen vertex, and so on. If there are still several, pick the one connected to the highest number of unchosen neighbors of the lowest-numbered chosen vertex. If there are several, it must be the one connected to



$$\begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Fig. 5. Connection matrices corresponding to two vertex labelings of a simple graph.

the highest number of unchosen neighbors of the next-lowest-numbered chosen vertex, etc. (The distinguishing feature of this algorithm is that it stops after examining the chosen vertices or their nearest neighbors only.)

2. If there are still several candidate vertices, divide ps by the number of such candidate vertices.

3. If there are still unchosen vertices, go to step 1. If not, the graph matrix is acceptable and its pseudo-symmetry number is the value of ps.

As an example, consider the graph shown in Fig. 6. Its first vertex is unique (the one of degree 4). The next vertex can be chosen in four

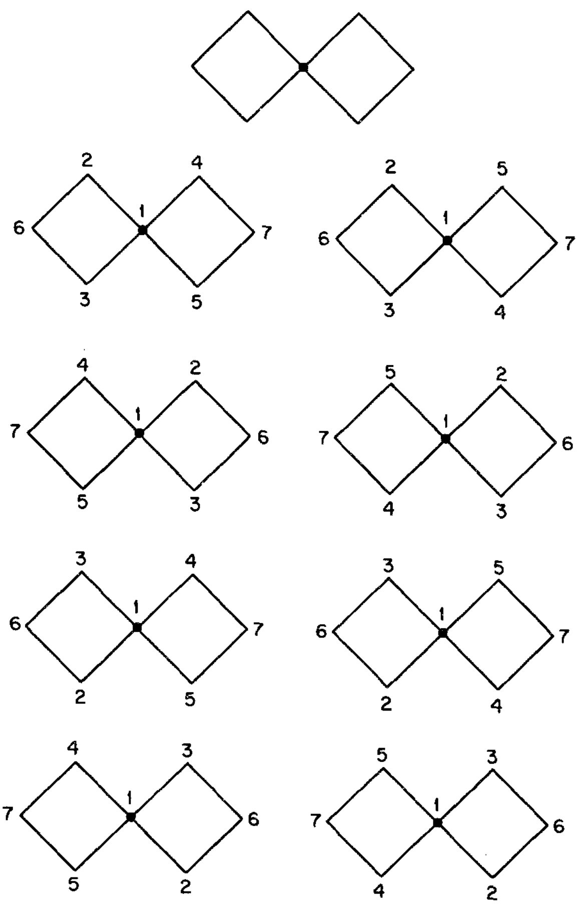


Fig. 6. Vertex labelings consistent with the algorithm given in the text. Since there are eight such labelings, the connection matrix associated with these labelings would have associated with it a pseudo-symmetry number of 1/8.

equivalent ways. The position of the third vertex is then uniquely determined by the rules of step 1. Having chosen vertices 2 and 3 as shown in the first column of the figure, we can choose the fourth vertex in two ways. The choices for the remaining vertices are completely determined by the algorithm. The pseudo-symmetry number for this graph is 1/8, which coincides with the true symmetry number.

As a second example, consider the graph shown in Fig. 7. It will appear as three different acceptable connection matrices, corresponding to the vertex labelings shown. The corresponding pseudo-symmetry numbers for the labelings shown in Fig. 7 are 1/8, 1/24, 1/12. The true symmetry number is $1/4 = 1/8 + 1/24 + 1/12$.

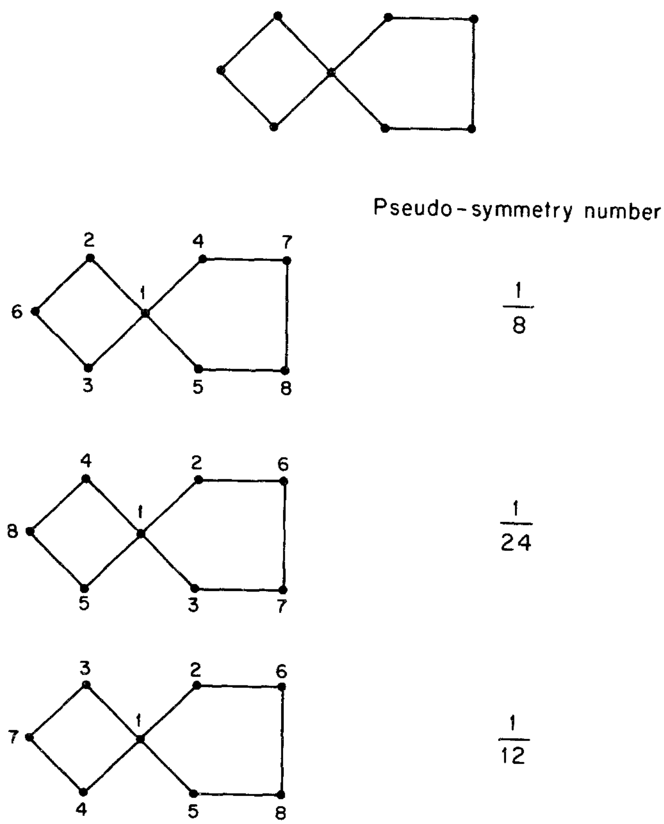


Fig. 7. An example of labelings for each of three acceptable connection matrices associated with the graph shown. According to the algorithm given in the text, each matrix would be accepted with the pseudo-symmetry number shown, which sum to the true symmetry number of the graph.

Table I. Strong Coupling Expansion of the Susceptibility χ through Fifteenth Order[a]

Partition	Linear Chain (LC)	Plane Square (PSQ)	Simple Cubic (SC)	Body-Centered Cubic (BCC)
- 0 -				
(1,0,0,0,0,0,0,0)	1	1	1	1
- 1 -				
(2,0,0,0,0,0,0,0)	2	4	6	8
- 2 -				
(3,0,0,0,0,0,0,0)	2	20	54	104
(1,1,0,0,0,0,0,0)	2	4	6	8
- 3 -				
(4,0,0,0,0,0,0,0)	-6	132	702	1992
(2,1,0,0,0,0,0,0)	12	72	180	336
(0,2,0,0,0,0,0,0)	2	4	6	8
- 4 -				
(5,0,0,0,0,0,0,0)	-12	1032	11772	48336
(3,1,0,0,0,0,0,0)	-6	972	4662	13368
(2,0,1,0,0,0,0,0)	6	36	90	168
(1,2,0,0,0,0,0,0)	26	164	414	776
(0,1,1,0,0,0,0,0)	2	4	6	8
- 5 -				
(6,0,0,0,0,0,0,0)	240	10560	248400	1464000
(4,1,0,0,0,0,0,0)	-480	11760	127440	558240
(3,0,1,0,0,0,0,0)	0	720	3600	10080
(2,2,0,0,0,0,0,0)	210	5460	22950	62760
(1,1,1,0,0,0,0,0)	60	360	900	1680
(0,0,2,0,0,0,0,0)	2	4	6	8
- 6 -				
(7,0,0,0,0,0,0,0)	720	132480	6162480	52430400
(5,1,0,0,0,0,0,0)	-360	154440	3931200	25328880
(4,0,1,0,0,0,0,0)	-360	11160	128880	602640

(3,2,0,0,0,0,0,0)	-780	116760	1013580	4321200
(3,0,0,1,0,0,0,0)	0	360	1800	5040
(2,1,1,0,0,0,0,0)	360	14400	64440	178560
(1,3,0,0,0,0,0,0)	330	9540	37710	109320
(1,1,0,1,0,0,0,0)	30	180	450	840
(1,0,2,0,0,0,0,0)	52	320	804	1504
(0,2,1,0,0,0,0,0)	70	420	1050	1960
(0,0,1,1,0,0,0,0)	2	4	6	8
- 7 -				
(8,0,0,0,0,0,0,0)	-16380	1917720	178230780	2191185360
(6,1,0,0,0,0,0,0)	40320	2439360	135898560	1274353920
(5,0,1,0,0,0,0,0)	0	100800	4415040	33183360
(4,2,0,0,0,0,0,0)	-28140	2221800	44963100	295100400
(4,0,0,1,0,0,0,0)	0	5040	75600	352800
(3,1,1,0,0,0,0,0)	-4200	438480	3767400	16070880
(2,3,0,0,0,0,0,0)	5040	537600	3996720	16813440
(2,1,0,1,0,0,0,0)	0	10080	50400	141120
(2,0,2,0,0,0,0,0)	924	20552	85092	230832
(1,2,1,0,0,0,0,0)	2240	47040	188160	519680
(1,0,1,1,0,0,0,0)	112	672	1680	3136
(0,4,0,0,0,0,0,0)	70	12460	43890	151480
(0,2,0,1,0,0,0,0)	140	840	2100	3920
(0,0,0,2,0,0,0,0)	2	4	6	8
- 8 -				
(9,0,0,0,0,0,0,0)	-65520	29161440	5853118320	103776644160
(7,1,0,0,0,0,0,0)	63000	47612880	5174948520	70672704480
(6,0,1,0,0,0,0,0)	37800	1154160	174628440	1970831520
(5,2,0,0,0,0,0,0)	63000	41277600	2069648280	20325614400
(5,0,0,1,0,0,0,0)	0	17640	3107160	27684720
(4,1,1,0,0,0,0,0)	-59640	9176160	189108360	1291281600
(4,0,0,0,1,0,0,0)	0	2520	37800	176400
(3,3,0,0,0,0,0,0)	-66360	19078080	303516360	1989657600
(3,1,0,1,0,0,0,0)	-3360	399840	3931200	17424960
(3,0,2,0,0,0,0,0)	-1792	684992	5668992	23816576
(2,2,1,0,0,0,0,0)	11480	3456320	26412120	115673600
(2,1,0,0,1,0,0,0)	0	5040	25200	70560
(2,0,1,1,0,0,0,0)	1400	46592	206136	566720
(1,4,0,0,0,0,0,0)	11060	1246280	8927100	41667920
(1,2,0,1,0,0,0,0)	1750	75460	342090	949480
(1,1,2,0,0,0,0,0)	5264	122080	480816	1362368
(1,0,1,0,1,0,0,0)	56	336	840	1568
(1,0,0,2,0,0,0,0)	86	524	1314	2456
(0,3,1,0,0,0,0,0)	1540	83720	293580	980560
(0,2,0,0,1,0,0,0)	70	420	1050	1960
(0,1,1,1,0,0,0,0)	420	2520	6300	11760
(0,0,0,1,1,0,0,0)	2	4	6	8

[a] The results are completely general for any even potential in zero external field for the lattice geometries linear chain (LC), plane square (PSQ), simple cubic (SC), and body-centered cubic (BCC). The partitions are grouped by perturbation order and the corresponding coefficients in each nth order have been multiplied by $n!$ for convenience.

(Table continued)

Table I. *(Continued)*

- 9 -				
(10,0,0,0,0,0,0,0,0)	2086560	489706560	216250443360	5527262240640
(8,1,0,0,0,0,0,0,0)	-6168960	979594560	216899000640	4309745408640
(7,0,1,0,0,0,0,0,0)	0	28486080	7401119040	123195219840
(6,2,0,0,0,0,0,0,0)	5995080	919734480	101944354680	1477552497120
(6,0,0,1,0,0,0,0,0)	0	-1088640	115123680	1776297600
(5,1,1,0,0,0,0,0,0)	589680	154738080	9534445200	102238718400
(5,0,0,0,1,0,0,0,0)	0	0	1360800	12700800
(4,3,0,0,0,0,0,0,0)	-2192400	511025760	20411591760	206833193280
(4,1,0,1,0,0,0,0,0)	0	8074080	230519520	1692290880
(4,0,2,0,0,0,0,0,0)	-153720	20215440	352651320	2341715040
(3,2,1,0,0,0,0,0,0)	-594720	152449920	2388022560	15969542400
(3,1,0,0,1,0,0,0,0)	0	151200	2268000	10584000
(3,0,1,1,0,0,0,0,0)	-12096	2356704	19613664	82555200
(2,4,0,0,0,0,0,0,0)	246960	103602240	1354041360	8918159040
(2,2,0,1,0,0,0,0,0)	-15120	5508720	44180640	189332640
(2,1,2,0,0,0,0,0,0)	97776	9658656	70425936	299113920
(2,0,1,0,1,0,0,0,0)	0	30240	151200	423360
(2,0,0,2,0,0,0,0,0)	2682	55908	230526	621576
(1,3,1,0,0,0,0,0,0)	85680	12902400	90039600	446019840
(1,2,0,0,1,0,0,0,0)	0	75600	378000	1058400
(1,1,1,1,0,0,0,0,0)	21672	454608	1818936	5021856
(1,0,0,1,1,0,0,0,0)	180	1080	2700	5040
(0,2,2,0,0,0,0,0,0)	10416	508704	1775088	5916288
(0,1,1,0,1,0,0,0,0)	840	5040	12600	23520
(0,0,0,0,2,0,0,0,0)	2	4	6	8
- 10 -				
(11,0,0,0,0,0,0,0,0)	10432800	9662587200	8825075128800	325382704214400
(9,1,0,0,0,0,0,0,0)	-15876000	19951596000	9932152608000	286074500241600
(8,0,1,0,0,0,0,0,0)	-6350400	885880800	343399240800	8419005604800
(7,2,0,0,0,0,0,0,0)	-3780000	23791622400	5316170680800	113290264483200
(7,0,0,1,0,0,0,0,0)	0	-17690400	5407138800	127421683200
(6,1,1,0,0,0,0,0,0)	13456800	2963822400	503085945600	8182030752000
(6,0,0,0,1,0,0,0,0)	0	-1814400	48308400	1191153600
(5,3,0,0,0,0,0,0,0)	13381200	13248295200	1345723016400	20593050811200
(5,1,0,1,0,0,0,0,0)	567000	102135600	12670295400	155281341600
(5,0,2,0,0,0,0,0,0)	181440	415265760	19520948160	206630827200
(5,0,0,0,0,1,0,0,0)	0	0	680400	6350400
(4,2,1,0,0,0,0,0,0)	-6400800	4588567200	180779256000	1923859828800
(4,1,0,0,1,0,0,0,0)	0	4384800	185749200	1499299200
(4,0,1,1,0,0,0,0,0)	-284760	70716240	1356558840	9211839840
(3,4,0,0,0,0,0,0,0)	-4901400	4863978000	143381107800	1455184836000
(3,2,0,1,0,0,0,0,0)	-529200	240798600	4338646200	30390519600
(3,1,2,0,0,0,0,0,0)	-1267560	533589840	8032658760	54959879520
(3,1,0,0,0,1,0,0,0)	0	75600	1134000	5292000
(3,0,1,0,1,0,0,0,0)	-10080	1975680	18779040	81849600
(3,0,0,2,0,0,0,0,0)	-5220	2837880	23264820	96989040
(2,3,1,0,0,0,0,0,0)	88200	1132588800	15636537000	111954124800
(2,2,0,0,1,0,0,0,0)	-12600	5304600	51710400	228186000
(2,1,1,1,0,0,0,0,0)	151200	43626240	332942400	1438819200
(2,0,3,0,0,0,0,0,0)	83160	7817040	55467720	249994080
(2,0,1,0,0,1,0,0,0)	0	15120	75600	211680
(2,0,0,1,1,0,0,0,0)	3780	115200	507060	1386720
(1,5,0,0,0,0,0,0,0)	352800	208303200	2506442400	18295401600
(1,3,0,1,0,0,0,0,0)	37800	21798000	156907800	771271200
(1,2,2,0,0,0,0,0,0)	482160	65730000	452571840	2294916960
(1,2,0,0,0,1,0,0,0)	0	37800	189000	529200
(1,1,1,0,1,0,0,0,0)	15960	670320	3031560	8410080

(1,1,0,2,0,0,0,0)	22590	496620	1970730	5511960
(1,0,2,1,0,0,0,0)	18900	430920	1689660	4793040
(1,0,0,1,0,1,0,0)	90	540	1350	2520
(1,0,0,0,2,0,0,0)	128	776	1944	3632
(0,4,1,0,0,0,0,0)	96600	14044800	95823000	552316800
(0,3,0,0,1,0,0,0)	0	138600	693000	1940400
(0,2,1,1,0,0,0,0)	34020	1317960	4698540	15155280
(0,1,3,0,0,0,0,0)	10080	927360	3024000	10946880
(0,1,1,0,0,1,0,0)	420	2520	6300	11760
(0,1,0,1,1,0,0,0)	990	5940	14850	27720
(0,0,2,0,1,0,0,0)	924	5544	13860	25872
(0,0,0,0,1,1,0,0)	2	4	6	8

– 11 –

(12,0,0,0,0,0,0,0)	-414136800	210561120000	395971387812000	21063331312531200
(10,1,0,0,0,0,0,0)	1427025600	439124716800	494025634872000	20604007457510400
(9,0,1,0,0,0,0,0)	0	20756736000	17104907635200	615607347801600
(8,2,0,0,0,0,0,0)	-1751349600	632531592000	296248583584800	9250402378694400
(8,0,0,1,0,0,0,0)	0	548856000	265247136000	9226947945600
(7,1,1,0,0,0,0,0)	-133056000	85727980800	28145694931200	682674663686400
(7,0,0,0,1,0,0,0)	0	0	1706443200	80911353600
(6,3,0,0,0,0,0,0)	931392000	388802937600	90327267676800	2057966381116800
(6,1,0,1,0,0,0,0)	0	209563200	687162722400	13586960217600
(6,0,2,0,0,0,0,0)	37477440	7886672640	1121104575360	18541571368320
(6,0,0,0,0,1,0,0)	0	0	14968800	419126400
(5,2,1,0,0,0,0,0)	200138400	105664204800	12820436244000	213640723296000
(5,1,0,0,1,0,0,0)	0	-43243200	9879408000	148789872000
(5,0,1,1,0,0,0,0)	2550240	1569173760	87761409120	976482904320
(4,4,0,0,0,0,0,0)	-205474500	178994277000	13341593103300	208870859458800
(4,2,0,1,0,0,0,0)	3187800	7060006800	353815446600	4015363413600
(4,1,2,0,0,0,0,0)	-44352000	20272412160	709464571200	7616516181120
(4,1,0,0,0,1,0,0)	0	0	74844000	698544000
(4,0,1,0,1,0,0,0)	0	60984000	1606651200	11603813760
(4,0,0,2,0,0,0,0)	-710820	125480520	2107209060	13925268720
(3,3,1,0,0,0,0,0)	-73735200	67559184000	1977493240800	21166033996800
(3,2,0,0,1,0,0,0)	0	220096800	5513508000	40518878400
(3,1,1,1,0,0,0,0)	-5987520	2677973760	40190230080	270489542400
(3,0,3,0,0,0,0,0)	0	500068800	7445148480	50682360960
(3,0,1,0,0,1,0,0)	0	665280	9979200	46569600
(3,0,0,1,1,0,0,0)	-27720	8814960	72527400	302021280
(2,5,0,0,0,0,0,0)	12889800	23934279600	575979881400	5954560365600
(2,3,0,1,0,0,0,0)	-1524600	2096186400	31334272200	230917579200
(2,2,2,0,0,0,0,0)	5118960	6658417920	87217043760	654713431680
(2,2,0,0,0,1,0,0)	0	2494800	37422000	174636000
(2,1,1,0,1,0,0,0)	-129360	73365600	584614800	2494282560
(2,1,0,2,0,0,0,0)	451440	54719280	404537760	1719701280
(2,0,2,1,0,0,0,0)	526680	47401200	340872840	1462396320
(2,0,0,1,0,1,0,0)	0	71280	356400	997920
(2,0,0,0,2,0,0,0)	6204	124872	514404	1380720
(1,4,1,0,0,0,0,0)	6006000	3542800800	41831974800	321296606400
(1,3,0,0,1,0,0,0)	0	29013600	224532000	972417600
(1,2,1,1,0,0,0,0)	1533840	268588320	1857628080	9309854400
(1,1,3,0,0,0,0,0)	1201200	181288800	1198002960	6673128000
(1,1,1,0,0,1,0,0)	0	665280	3326400	9313920
(1,1,0,1,1,0,0,0)	77880	1613040	6474600	17798880
(1,0,2,0,1,0,0,0)	66528	1419264	5654880	15700608
(1,0,0,0,1,1,0,0)	264	1584	3960	7392

(Table continued)

Table I. *(Continued)*

(0,6,0,0,0,0,0,0)	138600	275629200	3162159000	28101981600
(0,4,0,1,0,0,0,0)	-92400	55347600	367567200	2388540000
(0,3,2,0,0,0,0,0)	600600	46292400	315592200	1959434400
(0,3,0,0,0,1,0,0)	0	277200	1386000	3880800
(0,2,0,2,0,0,0,0)	90090	3245220	11698830	37188360
(0,1,2,1,0,0,0,0)	92400	4952640	16798320	57583680
(0,1,0,1,0,1,0,0)	1980	11880	29700	55440
(0,0,4,0,0,0,0,0)	2772	1768536	5430348	21211344
(0,0,2,0,0,1,0,0)	1848	11088	27720	51744
(0,0,0,0,0,2,0,0)	2	4	6	8

- 12 -

(13,0,0,0,0,0,0,0,0)	-2484820800	4759120396800	19319725082750400	148229749970184960O
(11,1,0,0,0,0,0,0,0)	5074423200	11269770019200	26452839574576800	1597172602735987200
(10,0,1,0,0,0,0,0,0)	1571724000	331049980800	924851394482400	48567864040819200
(9,2,0,0,0,0,0,0,0)	-953013600	16409417270400	17568729390088800	800382489369724800
(9,0,0,1,0,0,0,0,0)	0	16720149600	14230119657600	740491297761600
(8,1,1,0,0,0,0,0,0)	-4206232800	2805662059200	1664462586679200	59841615820694400
(8,0,0,0,1,0,0,0,0)	0	1122660000	96099696000	6561393854400
(7,3,0,0,0,0,0,0,0)	-3582532800	13038817730400	6225622519485600	20968751502578880O
(7,1,0,1,0,0,0,0,0)	-139708800	29807870400	40517697528000	1226642904748800
(7,0,2,0,0,0,0,0,0)	-11975040	200941836480	66356732312640	1661880100446720
(7,0,0,0,0,1,0,0,0)	0	0	-187110000	42301828800
(6,2,1,0,0,0,0,0,0)	3170059200	2683604800800	912638889439200	23211006084604800
(6,1,0,0,1,0,0,0,0)	0	-3981700800	557221064400	15123647246400
(6,0,1,1,0,0,0,0,0)	77006160	26972779680	5339071584000	95410519142400
(6,0,0,0,0,0,1,0,0)	0	0	7484400	209563200
(5,4,0,0,0,0,0,0,0)	2449062000	5935047703200	1169103568525200	27847339959700800
(5,2,0,1,0,0,0,0,0)	195218100	133651841400	26292998447100	488159743856400
(5,1,2,0,0,0,0,0,0)	366236640	549227558880	55967951017440	962608568866560
(5,1,0,0,0,1,0,0,0)	0	-79833600	5762988000	110230243200
(5,0,1,0,1,0,0,0,0)	2494800	1211807520	114871070160	1410629774400
(5,0,0,2,0,0,0,0,0)	653400	3194318160	148406777640	1609654701600
(4,3,1,0,0,0,0,0,0)	-597504600	2716728260400	202524779179800	3390249767335200
(4,2,0,0,1,0,0,0,0)	3118500	5928060600	482673321900	6060480426000
(4,1,1,1,0,0,0,0,0)	-78586200	103527048240	3868504489080	42864393227040
(4,1,0,0,0,0,1,0,0)	0	0	37422000	349272000
(4,0,3,0,0,0,0,0,0)	-39916800	25575026400	866839216320	9748013869440
(4,0,1,0,0,1,0,0,0)	0	34927200	1251890640	9776954880
(4,0,0,1,1,0,0,0,0)	-1128600	372604320	7012490760	47341514880
(3,5,0,0,0,0,0,0,0)	-492930900	1622161787400	88092067209300	1400848506010800
(3,3,0,1,0,0,0,0,0)	-56964600	130280119200	4478132320200	51822241732800
(3,2,2,0,0,0,0,0,0)	-312404400	431614814400	12337016685840	140536959615360
(3,2,0,0,0,1,0,0,0)	0	139293000	4888560600	38583745200
(3,1,1,0,1,0,0,0,0)	-5405400	4128006960	73503543960	511751671200
(3,1,0,2,0,0,0,0,0)	-6557760	4018564440	60543699480	413334401040
(3,0,2,1,0,0,0,0,0)	-5114340	4667022360	67915067220	474405679440
(3,0,1,0,0,0,1,0,0)	0	332640	4989600	23284800
(3,0,0,1,0,1,0,0,0)	-23760	6961680	65007360	279940320
(3,0,0,0,2,0,0,0,0)	-12144	9032496	73827072	305438496
(2,4,1,0,0,0,0,0,0)	-34303500	437283415800	11026501447500	126294529006800
(2,3,0,0,1,0,0,0,0)	-1039500	2982394800	53246308500	402472778400
(2,2,1,1,0,0,0,0,0)	-2730420	32787049680	450847989900	3456845330400
(2,2,0,0,0,0,1,0,0)	0	1247400	18711000	87318000
(2,1,3,0,0,0,0,0,0)	16437960	17947535760	235855148760	1931769433440
(2,1,1,0,0,1,0,0,0)	-110880	66472560	641496240	2818680480
(2,1,0,1,1,0,0,0,0)	625680	220809600	1698388560	7279430400

(2,0,2,0,1,0,0,0,0)	496188	151900056	1159208820	4990852944
(2,0,1,2,0,0,0,0,0)	1235520	124704360	890536680	3989280240
(2,0,0,1,0,0,1,0)	0	35640	178200	498960
(2,0,0,0,1,1,0,0)	8316	240768	1056924	2879712
(1,6,0,0,0,0,0,0)	24948000	55328565600	1288582495200	15168962793600
(1,4,0,1,0,0,0,0)	1247400	8994724200	121722123600	1046925356400
(1,3,2,0,0,0,0,0)	36327060	30209727240	359514526140	3042012702960
(1,3,0,0,0,1,0,0)	0	33818400	356479200	1595563200
(1,2,1,0,1,0,0,0)	803880	490865760	3565817640	17063323200
(1,2,0,2,0,0,0,0)	2722500	467569080	3277329660	16043520240
(1,1,2,1,0,0,0,0)	7151760	1179097920	7831509840	43606442880
(1,1,1,0,0,0,1,0)	0	332640	1663200	4656960
(1,1,0,1,0,1,0,0)	54450	2211660	9988110	27660600
(1,1,0,0,2,0,0,0)	73524	1561032	6236604	17268240
(1,0,4,0,0,0,0,0)	753984	220651200	1399749120	8727941376
(1,0,2,0,0,1,0,0)	47124	2012472	9089388	25269552
(1,0,1,1,1,0,0,0)	187308	4207896	16557156	46748592
(1,0,0,0,1,0,1,0)	132	792	1980	3696
(1,0,0,0,0,2,0,0)	178	1076	2694	5032
(0,5,1,0,0,0,0,0)	3880800	5128754400	56300983200	501902755200
(0,4,0,0,1,0,0,0)	-138600	65280600	436590000	2691334800
(0,3,1,1,0,0,0,0)	2106720	619930080	4083488640	26871768000
(0,3,0,0,0,0,1,0)	0	138600	693000	1940400
(0,2,3,0,0,0,0,0)	3404940	350944440	2344238820	16169168400
(0,2,1,0,0,1,0,0)	0	2522520	12612600	35315280
(0,2,0,1,1,0,0,0)	218790	7050780	25818210	80281080
(0,1,2,0,1,0,0,0)	129360	5510736	19248768	63541632
(0,1,1,2,0,0,0,0)	248490	15394500	51473070	179811720
(0,1,0,1,0,0,1,0)	990	5940	14850	27720
(0,1,0,0,1,1,0,0)	2002	12012	30030	56056
(0,0,3,1,0,0,0,0)	44352	11764368	36357552	140374080
(0,0,2,0,0,0,1,0)	924	5544	13860	25872
(0,0,1,1,0,1,0,0)	6006	36036	90090	168168
(0,0,0,0,0,1,1,0)	2	4	6	8

- 13 -

(14,0,0,0,0,0,0,0)	118702584000	114990501225600	1020958352835220800	112976366215218105600
(12,1,0,0,0,0,0,0)	-467804937600	315412614316800	1521115801520515200	132913731152903500800
(11,0,1,0,0,0,0,0)	0	5230697472000	53414355407212800	4085206579956902400
(10,2,0,0,0,0,0,0)	693599306400	449143464369600	1108406255938581600	73517660747000870400
(10,0,0,1,0,0,0,0)	0	-322248326400	805044038198400	62282883100838400
(9,1,1,0,0,0,0,0)	42681038400	80666643657600	104328959324990400	5532895399596364800
(9,0,0,0,1,0,0,0)	0	0	5550610665600	526080511756800
(8,3,0,0,0,0,0,0)	-485707622400	438561810086400	445997930217840000	22108563692062848000
(8,1,0,1,0,0,0,0)	0	3285531849600	2508030702777600	113621939048217600
(8,0,2,0,0,0,0,0)	-12363230880	7454115789120	4178082615426720	156126379242113280
(8,0,0,0,0,1,0,0)	0	0	0	2772581011200
(7,2,1,0,0,0,0,0)	-85448563200	98292139545600	66534779302992000	2538396026271705600
(7,1,0,0,1,0,0,0)	0	37621584000	31391968608000	1396366344172800
(7,0,1,1,0,0,0,0)	-795674880	570152943360	338266870381440	9450367267461120
(7,0,0,0,0,0,1,0)	0	0	0	10897286400
(6,4,0,0,0,0,0,0)	164907943200	219259888848000	101659391724290400	3631562422747459200
(6,2,0,1,0,0,0,0)	-994593600	1790938749600	1903837079944800	56354052575620800
(6,1,2,0,0,0,0,0)	20964303360	11884303791360	4265592845754240	115056744480437760
(6,1,0,0,0,1,0,0)	0	0	227480853600	10435189564800
(6,0,1,0,1,0,0,0)	0	-4428103680	7350219676800	155856207386880
(6,0,0,2,0,0,0,0)	242702460	60937236360	10343893980420	184231843876080

(Table continued)

Table I. *(Continued)*

(5,3,1,0,0,0,0,0)	51372921600	83395505793600	1905254116365600	496564401770236800
(5,2,0,0,1,0,0,0)	0	-23113490400	35611813036800	780126744196800
(5,1,1,1,0,0,0,0)	2337294960	2896935481440	330207230873520	5931419357223360
(5,1,0,0,0,0,1,0)	0	0	1362160800	38140502400
(5,0,3,0,0,0,0,0)	0	775056522240	77702166622080	1398411401587200
(5,0,1,0,0,1,0,0)	0	-242161920	88903694880	1314679766400
(5,0,0,1,1,0,0,0)	8185320	11113193520	603780554520	6797033511840
(4,5,0,0,0,0,0,0)	-24518894400	77055728349600	11015583254676000	269839042376904000
(4,3,0,1,0,0,0,0)	918918000	4987540958400	498938257818000	9139947616800000
(4,2,2,0,0,0,0,0)	-7230983760	21137087331360	1441576951935120	25255155386260800
(4,2,0,0,0,1,0,0)	0	-908107200	411826615200	6192426240000
(4,1,1,0,1,0,0,0)	38198160	164844519840	7718251741200	88721984070720
(4,1,0,2,0,0,0,0)	-282779640	201889836720	6845766590520	74654725036320
(4,0,2,1,0,0,0,0)	-239999760	241150029120	7896595346640	86842654778880
(4,0,1,0,0,0,1,0)	0	0	454053600	4237833600
(4,0,0,1,0,1,0,0)	0	304864560	7751343600	55216006560
(4,0,0,0,2,0,0,0)	-2376660	568754472	9368716500	61260753840
(3,4,1,0,0,0,0,0)	-10061251200	34634206807200	1885286049847200	32092365586881600
(3,3,0,0,1,0,0,0)	0	170522352000	7756057108800	94295689488000
(3,2,1,1,0,0,0,0)	-1327566240	2415570917760	71484386893920	839339562821760
(3,2,0,0,0,0,1,0)	0	0	2270268000	21189168000
(3,1,3,0,0,0,0,0)	-863422560	1400543464320	37867598889120	463722871772160
(3,1,1,0,0,1,0,0)	0	3904860960	95442066720	696466330560
(3,1,0,1,1,0,0,0)	-29446560	18304331760	277432000560	1867564052640
(3,0,2,0,1,0,0,0)	-20324304	12998905920	185143598640	1245196592640
(3,0,1,2,0,0,0,0)	0	10164314160	153774981360	1058754771360
(3,0,0,1,0,0,1,0)	0	2162160	32432400	151351200
(3,0,0,0,1,1,0,0)	-54912	26090064	213765552	882806496
(2,6,0,0,0,0,0,0)	1099998900	8725236319800	377108317485900	6201913398080400
(2,4,0,1,0,0,0,0)	-102702600	1147908762000	30963055523400	381260094415200
(2,3,2,0,0,0,0,0)	-123243120	3986123581440	95980148344080	1168750935031680
(2,3,0,0,0,1,0,0)	0	3153150000	74086412400	548590442400
(2,2,1,0,1,0,0,0)	-18018000	59429129760	881168848560	6542122466880
(2,2,0,2,0,0,0,0)	35727120	60206065920	782510105520	5926008179520
(2,1,2,1,0,0,0,0)	59099040	146322015840	1948382916480	16479052349760
(2,1,1,0,0,0,1,0)	0	30270240	454053600	2118916800
(2,1,0,1,0,1,0,0)	-411840	343783440	2739456720	11599267680
(2,1,0,0,2,0,0,0)	1513512	231728640	1739560680	7372430208
(2,0,4,0,0,0,0,0)	50834784	27999299328	340691551200	3162211852800
(2,0,2,0,0,1,0,0)	-384384	304720416	2411529120	10313983680
(2,0,1,1,1,0,0,0)	6321744	617945328	4470482016	19235028384
(2,0,0,0,1,0,1,0)	0	144144	720720	2018016
(2,0,0,0,0,2,0,0)	12402	244244	1006278	2691624
(1,5,1,0,0,0,0,0)	448648200	1402676074800	32333875774200	415667454602400
(1,4,0,0,1,0,0,0)	0	13437824400	225113288400	2142895154400
(1,3,1,1,0,0,0,0)	74234160	184670806320	2225225882880	20164415150880
(1,3,0,0,0,0,1,0)	0	25225200	378378000	1765764000
(1,2,3,0,0,0,0,0)	251531280	114369615360	1300156537680	11888680003200
(1,2,1,0,0,1,0,0)	0	731530800	5665579920	24511687200
(1,2,0,1,1,0,0,0)	8288280	1935236160	13666550040	66825982080
(1,1,2,0,1,0,0,0)	6870864	1503614112	10201215024	53217772608
(1,1,1,2,0,0,0,0)	24967800	3956752800	26219433240	144758877120
(1,1,0,1,0,0,1,0)	0	2162160	10810800	30270240
(1,1,0,0,1,1,0,0)	227084	4636632	18678660	51097904
(1,0,3,1,0,0,0,0)	8024016	2678676000	16799550768	108232348224
(1,0,2,0,0,0,1,0)	0	2018016	10090080	28252224
(1,0,1,1,0,1,0,0)	598884	12756744	50846796	141103248
(1,0,0,0,0,1,1,0)	364	2184	5460	10192
(0,4,2,0,0,0,0,0)	58798740	74807973240	810708919020	7826674936080
(0,3,1,0,1,0,0,0)	-960960	2266664400	14987372400	93832458720

(0,2,2,1,0,0,0,0)	26066040	3556993440	23261598360	184803178560
(0,2,1,0,0,0,1,0)	0	5045040	25225200	70630560
(0,2,0,0,2,0,0,0)	494494	14986972	55489434	170049880
(0,1,1,1,1,0,0,0)	1446588	68348280	235423188	791247600
(0,1,0,0,1,0,1,0)	4004	24024	60060	112112
(0,0,2,2,0,0,0,0)	335478	76984908	238432194	917397624
(0,0,1,1,0,0,1,0)	12012	72072	180180	336336
(0,0,0,0,0,0,2,0)	2	4	6	8
- 14 -				
(15,0,0,0,0,0,0,0)	830918088000	3051169359638400	5795632790865197280 0	9249987908796008774400
(13,1,0,0,0,0,0,0)	-2127695169600	8901290385187200	9338611580146648800 0	11799995753091171225600
(12,0,1,0,0,0,0,0)	-539415676800	2437940913408 00	329597562363189120 0	3668573303297622720 00
(11,2,0,0,0,0,0,0)	1155112358400	14095912390560000	7396090934297345280 0	7134643382298758726400
(11,0,0,1,0,0,0,0)	0	-1702156135680 0	4963010807795040 0	565494813064621440 0
(10,1,1,0,0,0,0,0)	1741749609600	1919651442508800	692977084163844480 0	5395878429551850240 00
(10,0,0,0,1,0,0,0)	0	-751912761600	3231345092976 00	479313873118080 00
(9,3,0,0,0,0,0,0)	1032971940000	14106510909691200	3328751639441788560 0	2416870164651310368000
(9,1,0,1,0,0,0,0)	47675628000	929275178832 00	164853517978749600	111387160417268736 00
(9,0,2,0,0,0,0,0)	-5145940800	264231951984000	275179357728475200	1522987932872165760 0
(9,0,0,0,0,1,0,0)	0	0	964409846400	233959290364800
(8,2,1,0,0,0,0,0)	-1782765784800	4031984162606400	500955520620194640 0	2841416466998421312 00
(8,1,0,0,1,0,0,0)	0	42245146944 00	20481940166688 00	138641854093142400
(8,0,1,1,0,0,0,0)	-27939431520	29127720061440	22379128033822560	9416358913664102 40
(8,0,0,0,0,0,1,0)	0	0	-24518894400	931717987200
(7,4,0,0,0,0,0,0)	-1291479789600	9054069432408000	886735884387553200 0	471153656142910540800
(7,2,0,1,0,0,0,0)	-89751261600	103365983120400	142766419103708400	650442018290304240 0
(7,1,2,0,0,0,0,0)	-139909049280	370687058501760	329112187782618240	13569711480774408960
(7,1,0,0,0,1,0,0)	0	68108040000	11049848409600	.11275658345952 00
(7,0,1,0,1,0,0,0)	-847566720	-239558679360	48288109982112 0	1664837930403648 0
(7,0,0,2,0,0,0,0)	-28108080	1362835393920	6970356424231 20	1959722167411008 0
(7,0,0,0,0,0,0,1)	0	0	0	5448643200
(6,3,1,0,0,0,0,0)	632648016000	2664447822036000	1739746327387068000	694004876329997232 00
(6,2,0,0,1,0,0,0)	-1059458400	-3416980366800	2611802350357200	9658659857006160 0
(6,1,1,1,0,0,0,0)	42741578880	58126700792160	26366064993908640	764873080185614400
(6,1,0,0,0,0,1,0)	0	0	49037788800	6378544972800
(6,0,3,0,0,0,0,0)	19917817920	16049402328960	6587609775626880	192182000492401920
(6,0,1,0,0,1,0,0)	0	-39109150080	6229615392000	171390583203840
(6,0,0,1,1,0,0,0)	421621200	187186839840	44063274855600	832299993645120
(5,5,0,0,0,0,0,0)	451329278400	3245779986649200	1257155936518482000	4701198500225987760 0
(5,3,0,1,0,0,0,0)	46313467200	127738596585600	49295968217496000	1450387535470473600
(5,2,2,0,0,0,0,0)	197563766400	730318769500320	147016778937668640	4058724522861328320
(5,2,0,0,0,1,0,0)	0	-174734960400	32990399442000	920141285263200
(5,1,1,0,1,0,0,0)	2320718400	3838074720480	6797237786539 20	13226739156671040
(5,1,0,2,0,0,0,0)	2246484240	6328369887840	643033769938560	11641438538009280
(5,1,0,0,0,0,0,1)	0	0	681080400	19070251200
(5,0,2,1,0,0,0,0)	1922160240	8290458736560	801931033743840	14853943282258080
(5,0,1,0,0,0,1,0)	0	-484323840	53941567680	956539584000
(5,0,0,1,0,1,0,0)	8108100	8569721160	732665753820	8971591028400
(5,0,0,0,2,0,0,0)	1873872	18077963904	848152665360	9295453335168
(4,4,1,0,0,0,0,0)	-34154920800	182013199968 6000	257139149141674800	6815481996842719200
(4,3,0,0,1,0,0,0)	794593800	5331169443600	906068915551800	18543758342709600
(4,2,1,1,0,0,0,0)	-8732964240	11954150135928 0	9078502238245440	169684993424026080
(4,2,0,0,0,0,1,0)	0	-1816214400	277880803200	4918914000000
(4,1,3,0,0,0,0,0)	-18545567040	74983975466400	4880073679921440	92179103493597120
(4,1,1,0,0,1,0,0)	37837800	140983642800	10459994945400	130983596583840
(4,1,0,1,1,0,0,0)	-444504060	895143249000	33768191637180	378171547593840

(*Table continued*)

Table I. *(Continued)*

(4,0,2,0,1,0,0,0)	-225008784	636420696912	22645505570688	254008555792992
(4,0,1,2,0,0,0,0)	-718377660	770021932680	25572245238540	292448116200240
(4,0,1,0,0,0,0,1)	0	0	227026800	2118916800
(4,0,0,1,0,0,1,0)	0	177297120	5837832000	44531847360
(4,0,0,0,1,1,0,0)	-3447444	1497487992	27916548660	186804041424
(3,6,0,0,0,0,0,0)	-55974718800	727856093764800	73438198140106800	1842086992044499200
(3,4,0,1,0,0,0,0)	-5770264500	85142630172600	5553283881845700	104908569458893200
(3,3,2,0,0,0,0,0)	-63453990600	346696874203680	18684665818018200	346338782188439040
(3,3,0,0,0,1,0,0)	0	148147599600	10816540534800	142848138232800
(3,2,1,0,1,0,0,0)	-1049368320	4595516845920	159859649869920	1947599813839680
(3,2,0,2,0,0,0,0)	-1901259360	4695183773280	136095288769440	1627240506091200
(3,2,0,0,0,0,0,1)	0	0	1135134000	10594584000
(3,1,2,1,0,0,0,0)	-5322517200	12666430576800	360388835286480	4631791228384320
(3,1,1,0,0,0,1,0)	0	2441799360	81366405120	634585311360
(3,1,0,1,0,1,0,0)	-24504480	25645379760	460959539040	3195950677920
(3,1,0,0,2,0,0,0)	-29129100	23068973928	353537772588	2405848356912
(3,0,4,0,0,0,0,0)	-444972528	2039885912064	53461016520912	731961727136640
(3,0,2,0,0,1,0,0)	-18666648	20639594976	362445259176	2516950275840
(3,0,1,1,1,0,0,0)	-62486424	73012251312	1064458395000	7391192608800
(3,0,0,3,0,0,0,0)	0	8048640600	118634475960	864645621840
(3,0,0,1,0,0,0,1)	0	1081080	16216200	75675600
(3,0,0,0,1,0,1,0)	-48048	19699680	182053872	777032256
(3,0,0,0,0,2,0,0)	-24388	23935184	195528060	803723648
(2,5,1,0,0,0,0,0)	-10039629600	219616821824400	10254551961654000	189690485199415200
(2,4,0,0,1,0,0,0)	-18918900	1840796357400	63505544602500	859291670437200
(2,3,1,1,0,0,0,0)	-2583060480	25527453391440	659063810124720	8737005185868960
(2,3,0,0,0,0,1,0)	0	2118916800	74464790400	592085894400
(2,2,3,0,0,0,0,0)	-234594360	19919529870240	473252053233960	6411833317169280
(2,2,1,0,0,1,0,0)	-12612600	89292162960	1614031899480	12069319822560
(2,2,0,1,1,0,0,0)	-12432420	278407289160	3908739334500	29616067681200
(2,1,2,0,1,0,0,0)	-6054048	327706963584	4484911406976	36682017660288
(2,1,1,2,0,0,0,0)	258918660	516008493000	6840147213900	59042702038320
(2,1,1,0,0,0,0,1)	0	15135120	227026800	1059458400
(2,1,0,1,0,0,1,0)	-360360	295495200	2831348520	12370438080
(2,1,0,0,1,1,0,0)	1937936	860411552	6677614944	28372408064
(2,0,3,1,0,0,0,0)	259146888	306059705952	3913663041288	38605652397312
(2,0,2,0,0,0,1,0)	-336336	266378112	2557835280	11247075840
(2,0,1,1,0,1,0,0)	5525520	1829667840	13996958976	60126690624
(2,0,1,0,2,0,0,0)	5921916	705272568	5105904804	22712241552
(2,0,0,2,1,0,0,0)	5549544	536720184	3819671856	17122721616
(2,0,0,0,1,0,0,1)	0	72072	360360	1009008
(2,0,0,0,0,1,1,0)	16016	448448	1965600	5340608
(1,7,0,0,0,0,0,0)	1709007300	18705557871000	739552117203900	13732712368174800
(1,5,0,1,0,0,0,0)	-44144100	3758390836200	91992413412900	1315680972606000
(1,4,2,0,0,0,0,0)	3801437640	18105893485680	414196767424440	5759408613198240
(1,4,0,0,0,1,0,0)	0	17653435800	344891547000	3110241934800
(1,3,1,0,1,0,0,0)	2522520	412810398000	5682337020360	53287760766240
(1,3,0,2,0,0,0,0)	168648480	320452652520	3806994030840	35139642698160
(1,3,0,0,0,0,0,1)	0	12612600	189189000	882882000
(1,2,2,1,0,0,0,0)	1123782660	1347687421080	16131970714860	165159328173840
(1,2,1,0,0,0,1,0)	0	813933120	8566477920	38342304000
(1,2,0,1,0,1,0,0)	4144140	3173450280	23385382020	109390160880
(1,2,0,0,2,0,0,0)	12292280	2791244456	19993925952	95372989712
(1,1,4,0,0,0,0,0)	558654096	294776978496	3280889403792	34729081330944
(1,1,2,0,0,1,0,0)	3867864	3901497600	28553412888	134691805248
(1,1,1,1,1,0,0,0)	91243152	17125700592	115216317216	619941145824
(1,1,0,3,0,0,0,0)	18558540	3236753520	20956195260	127885998240
(1,1,0,1,0,0,0,1)	0	1081080	5405400	15135120
(1,1,0,0,1,0,1,0)	152152	5989984	27027000	74698624
(1,1,0,0,0,2,0,0)	198016	4101552	16476096	45267040

(1,0,3,0,1,0,0,0)	11099088	3454170720	21868230384	141345876672
(1,0,2,2,0,0,0,0)	36732696	14194700520	88831622880	576721633488
(1,0,2,0,0,0,0,1)	0	1009008	5045040	14126112
(1,0,1,1,0,0,1,0)	408408	17297280	78053976	216984768
(1,0,1,0,1,1,0,0)	728728	16032016	63399336	177809632
(1,0,0,2,0,1,0,0)	546546	12336324	48486438	137104968
(1,0,0,0,0,1,0,1)	182	1092	2730	5096
(1,0,0,0,0,0,2,0)	236	1424	3564	6656
(0,6,1,0,0,0,0,0)	422522100	1474677804600	34663845516300	521337457040400
(0,5,0,0,1,0,0,0)	0	17796378600	305313208200	3651726078000
(0,4,1,1,0,0,0,0)	113933820	353827153680	3946155873660	41558311835040
(0,4,0,0,0,0,1,0)	0	63063000	945945000	4414410000
(0,3,3,0,0,0,0,0)	381741360	431170980240	4515764853600	47629862760480
(0,3,1,0,0,1,0,0)	-1681680	2398075680	16169353200	95405069760
(0,3,0,1,1,0,0,0)	7027020	5304859560	35554018500	212585733360
(0,2,2,0,1,0,0,0)	19255236	11986846872	76641472908	569585352336
(0,2,1,2,0,0,0,0)	79459380	11396745360	74397943620	593670036960
(0,2,1,0,0,0,0,1)	0	2522520	12612600	35315280
(0,2,0,1,0,0,1,0)	0	10810800	54054000	151351200
(0,2,0,0,1,1,0,0)	1013012	28804776	107831724	325493168
(0,1,3,1,0,0,0,0)	60540480	10740890160	68728579920	648358270560
(0,1,2,0,0,0,1,0)	0	20180160	100900800	282522240
(0,1,1,1,0,1,0,0)	1681680	67267200	237116880	773572800
(0,1,1,0,2,0,0,0)	1973972	91811720	317032716	1062357296
(0,1,0,2,1,0,0,0)	990990	65405340	217026810	765044280
(0,1,0,0,1,0,0,1)	2002	12012	30030	56056
(0,1,0,0,0,1,1,0)	3640	21840	54600	101920
(0,0,2,1,1,0,0,0)	1405404	212203992	666557892	2518628112
(0,0,1,3,0,0,0,0)	246246	158186028	480161682	1893595704
(0,0,1,1,0,0,0,1)	6006	36036	90090	168168
(0,0,1,0,1,0,1,0)	16016	96096	240240	448448
(0,0,0,2,0,0,1,0)	12870	77220	193050	360360
(0,0,0,0,0,0,1,1)	2	4	6	8

- 15 -

(16,0,0,0,0,0,0,0)	-46361142828000	87757740782712000	3524099786813121516000	811253876323181827536000
(14,1,0,0,0,0,0,0)	205713524016000	259192338429024000	6105575866003325712000	1115208617374277794368000
(13,0,1,0,0,0,0,0)	0	10727506677888000	215851778893086336000	34935261197346461952000
(12,2,0,0,0,0,0,0)	-357070021308000	480877483878792000	5219444795225120460000	731558559149309075184000
(12,0,0,1,0,0,0,0)	0	25826568768000	3218495230798848000	539413896401347968000
(11,1,1,0,0,0,0,0)	-18511765272000	49775344386768000	486248384461831992000	55437700234170814560000
(11,0,0,0,1,0,0,0)	0	0	19567058486976000	4524455237702400000
(10,3,0,0,0,0,0,0)	309564663408000	470462851915056000	2595197110455645696000	275275544996336268384000
(10,1,0,1,0,0,0,0)	0	29504402928000	11384157813668640000	1140766800464383072000
(10,0,2,0,0,0,0,0)	5408232429600	7612022442024000	19175667155665020000	1565335239591973449600
(10,0,0,0,0,1,0,0)	0	0	57619401840000	20835502623936000
(9,2,1,0,0,0,0,0)	46431521136000	142037019908784000	391455956741705856000	32824533203452999008000
(9,1,0,0,1,0,0,0)	0	-23728841136000	139302872588880000	13920186836556000000
(9,0,1,1,0,0,0,0)	337815878400	1389176989200000	1566339860772086400	97949525713252012800
(9,0,0,0,0,0,1,0)	0	0	0	53614649088000
(8,4,0,0,0,0,0,0)	-141871317588000	370184042788560000	789211327590802116000	62009431467154307136000
(8,2,0,1,0,0,0,0)	422269848000	7628981343984000	11052589376804976000	758485141364478528000
(8,1,2,0,0,0,0,0)	-11948874537600	18827988177398400	26122115205772185600	1617105636646346246400
(8,1,0,0,0,1,0,0)	0	0	748439251560000	105261738630048000
(8,0,1,0,1,0,0,0)	0	40100197737600	33133498974576000	1768582539134611200
(8,0,0,2,0,0,0,0)	-106897190400	59050254463200	49697651210407200	2148275111528145600
(8,0,0,0,0,0,0,1)	0	0	0	163459296000

(*Table continued*)

Table I. *(Continued)*

(7,3,1,0,0,0,0,0)	-39202987824000	118240415813520000	15931217404247745600 0	957717801043533129600 0
(7,2,0,0,1,0,0,0)	0	176635931472000	19179106918171200 0	113936060878960320 00
(7,1,1,1,0,0,0,0)	-1171458288000	1552027853376000	2109476080921809600	9606136741390949760 0
(7,1,0,0,0,0,1,0)	0	0	122594472000	522906287904000
(7,0,3,0,0,0,0,0)	0	198750158006400	536752178404041600	24685403944538937600
(7,0,1,0,0,1,0,0)	0	326918592000	372548254478400	1892185558623360 0
(7,0,0,1,1,0,0,0)	-3372969600	1100755656000	320540215675680 0	1004387955611040 00
(6,5,0,0,0,0,0,0)	33295750488000	147130547788224000	1381035250256341680 00	7784194981820152896000
(6,3,0,1,0,0,0,0)	-594810216000	2633992176816000	461561688210823200 0	214566555187176192000
(6,2,2,0,0,0,0,0)	6989095713600	19978794896860800	142699204259788176 00	614670413337620640000
(6,2,0,0,0,1,0,0)	0	1225944720000	2102495194800000	113288225738688000
(6,1,1,0,1,0,0,0)	-15740524800	17520414912000	55796005220155200	182448318706093440 0
(6,1,0,2,0,0,0,0)	178897118400	136442133828000	566192455861032 00	1671057521892158400
(6,1,0,0,0,0,0,1)	0	0	0	1525620096000
(6,0,2,1,0,0,0,0)	118962043200	167002730294400	70895366985643200	2149244709650438400
(6,0,1,0,0,0,1,0)	0	0	2574483912000	115565722272000
(6,0,0,1,0,1,0,0)	0	-72064792800	57242861676000	1264498930094400
(6,0,0,0,2,0,0,0)	1083962880	372766474080	73057081537440	1360725837831360
(5,4,1,0,0,0,0,0)	13249284048000	71143606261728000	31169572692882624000	1281374408796850368000
(5,3,0,0,1,0,0,0)	0	-6689723040000	8908073037864000 0	307372125751891200 0
(5,2,1,1,0,0,0,0)	1016474659200	4021676237779200	998067205175040000	297155390628096000 00
(5,2,0,0,0,0,1,0)	0	0	15038255232000	686529043200000
(5,1,3,0,0,0,0,0)	592691299200	3019065953904000	553757077809936000	163183102593166656 00
(5,1,1,0,0,1,0,0)	0	-673512840000	941228593905600	21280077400982400
(5,1,0,1,1,0,0,0)	14551336800	30726996300000	3542338079280000	65864743000056000
(5,0,2,0,1,0,0,0)	9081072000	23467306262400	2369781837783360	43857186880066560
(5,0,1,2,0,0,0,0)	0	30393180417600	2824450148520000	52276331031724800
(5,0,1,0,0,0,0,1)	0	0	10897286400	305124019200
(5,0,0,1,0,0,1,0)	0	-972972000	541167026400	7877829960000
(5,0,0,0,1,1,0,0)	21621600	59824084320	3156217384320	35431030514880
(4,6,0,0,0,0,0,0)	-3471996528000	43930835312364000	11775501537459660000	452834346950929944000
(4,4,0,1,0,0,0,0)	201297096000	4202372770596000	798067053499716000	23496890117140440000
(4,3,2,0,0,0,0,0)	-807912705600	21373817674809600	2830795036056187200	80051979967831872000
(4,3,0,0,0,1,0,0)	0	-784755972000	1205146794852000	28560278682936000
(4,2,1,0,1,0,0,0)	14227012800	220433639025600	21570315726960000	424327114613001600
(4,2,0,2,0,0,0,0)	-69497227800	274260277090800	19030334488565400	359310261727216800
(4,2,0,0,0,0,0,1)	0	0	68108040000	1907025120000
(4,1,2,1,0,0,0,0)	-77037760800	756284995065600	51601343187794400	1025392145979801600
(4,1,1,0,0,0,1,0)	0	-13621608000	9020228817600	134445270960000
(4,1,0,1,0,1,0,0)	162162000	1343631088800	628918398108000	7262829926352000
(4,1,0,0,2,0,0,0)	-1251169920	1474714200960	51252428747520	563597026312320
(4,0,4,0,0,0,0,0)	-50944813920	135265755905280	7741277002499040	159900739420181760
(4,0,2,0,0,1,0,0)	151351200	1118364287040	48880595443680	562976699725440
(4,0,1,1,1,0,0,0)	-3865942080	4900903207200	156296730189600	1725181712654400
(4,0,0,3,0,0,0,0)	0	478442764800	18256457419200	208487921241600
(4,0,0,1,0,0,0,1)	0	0	1945944000	18162144000
(4,0,0,0,1,0,1,0)	0	1163242080	29072403360	204808443840
(4,0,0,0,0,2,0,0)	-6459180	2052250200	33485519340	216575682960

(3,5,1,0,0,0,0,0)	-1670917248000	21973436621136000	2251044823547520000	61518411066176064000
(3,4,0,0,1,0,0,0)	0	128539547136000	12025167650496000	250892943917616000
(3,3,1,1,0,0,0,0)	-211286275200	2359721251087200	133818345986925600	2650137682011019200
(3,3,0,0,0,0,1,0)	0	-11351340000	10590800220000	160137137160000
(3,2,3,0,0,0,0,0)	-409354545600	1950915958992000	99307386573595200	1987560221590944000
(3,2,1,0,0,1,0,0)	0	6635488860000	292041372823200	3572514005860800
(3,2,0,1,1,0,0,0)	-9081072000	25184342383200	756310789634400	9043537464561600
(3,1,2,0,1,0,0,0)	-7002515520	29453589285120	891321080569920	11304644724172800
(3,1,1,2,0,0,0,0)	-17254036800	47930870988000	1340607848580000	17828333004081600
(3,1,1,0,0,0,0,1)	0	0	36324288000	339026688000
(3,1,0,1,0,0,1,0)	0	23481057600	567826459200	4102828329600
(3,1,0,0,1,1,0,0)	-114474360	93197504400	1435549915800	9626680583520
(3,0,3,1,0,0,0,0)	-6962155200	28937259711360	772043500388160	11450137462283520
(3,0,2,0,0,0,1,0)	0	20926825920	503273010240	3672506597760
(3,0,1,1,0,1,0,0)	-251170920	206938171440	2947291106760	19808056588320
(3,0,1,0,2,0,0,0)	0	70908757920	1111890780000	7720626513600
(3,0,0,2,1,0,0,0)	0	55973998080	838260783360	5803790952960
(3,0,0,0,1,0,0,1)	0	5765760	86486400	403603200
(3,0,0,0,0,1,1,0)	-98280	65541840	536313960	2200554720
(2,7,0,0,0,0,0,0)	102918816000	3671858814624000	293763325695840000	7689268449967104000
(2,5,0,1,0,0,0,0)	-15891876000	628078723272000	32359071965220000	656790030776880000
(2,4,2,0,0,0,0,0)	-87026940000	3034766370636000	137319282226526400	2763315399579134400
(2,4,0,0,0,1,0,0)	0	2052322272000	100000764864000	1412905830336000
(2,3,1,0,1,0,0,0)	-1513512000	59160812911200	1709351220376800	23986290435307200
(2,3,0,2,0,0,0,0)	4886481600	50089636396800	1247665736116800	16857474515481600
(2,3,0,0,0,0,0,1)	0	0	45405360000	423783360000
(2,2,2,1,0,0,0,0)	-28201773600	215374271112000	5174123755600800	75819252065356800
(2,2,1,0,0,0,1,0)	0	101960258400	2380603024800	17572177022400
(2,2,0,1,0,1,0,0)	-75675600	473275202400	7273192726800	53957437934400
(2,2,0,0,2,0,0,0)	62762700	459331072200	6137902510740	46164206328240
(2,1,4,0,0,0,0,0)	21148807680	50156536510080	1118697017518080	16553345724115200
(2,1,2,0,0,1,0,0)	-70630560	619349290560	8805885248160	64946817936000
(2,1,1,1,1,0,0,0)	1186665480	2683701100080	35531909332920	299068058409120
(2,1,0,3,0,0,0,0)	0	482723841600	6815085076800	63181169251200
(2,1,0,1,0,0,0,1)	0	129729600	1945944000	9081072000
(2,1,0,0,1,0,1,0)	-1081080	1264383120	10098368280	42428786400
(2,1,0,0,0,2,0,0)	4127760	797116320	6065382960	25585560000
(2,0,3,0,1,0,0,0)	0	463397014080	6699348976320	68441375882880
(2,0,2,2,0,0,0,0)	1765583820	2082138818760	25646778136980	263901302224560
(2,0,2,0,0,0,0,1)	0	121080960	1816214400	8475667200
(2,0,1,1,0,0,1,0)	-3243240	3510627120	27752404680	118326368160
(2,0,1,0,1,1,0,0)	27147120	3039996960	22230608400	95595339840
(2,0,0,2,0,1,0,0)	24504480	2335132800	16825929120	72763891200
(2,0,0,0,0,1,0,1)	0	262080	1310400	3669120
(2,0,0,0,0,0,2,0)	22380	434280	1790100	4775280
(1,6,1,0,0,0,0,0)	54486432000	672582534624000	26732357267616000	538593089082048000
(1,5,0,0,1,0,0,0)	0	7855127280000	214146812880000	3287372280192000

(Table continued)

Table I. *(Continued)*

(1,4,1,1,0,0,0,0)	4641436800	113695021440000	2628990322358400	39880726793107200
(1,4,0,0,0,0,1,0)	0	20684664000	467675208000	3496212720000
(1,3,3,0,0,0,0,0)	32187355200	134966221790400	3027841296480000	46810371149942400
(1,3,1,0,0,1,0,0)	0	567264297600	9810584784000	95178110227200
(1,3,0,1,1,0,0,0)	738738000	1867969303200	22316053359600	197654043787200
(1,2,2,0,1,0,0,0)	20180160	4080256820640	50740217367840	537652031716800
(1,2,1,2,0,0,0,0)	4306302000	4266294832800	50543073781200	554996994552000
(1,2,1,0,0,0,0,1)	0	605404800	9081072000	42378336000
(1,2,0,1,0,0,1,0)	0	4252248000	33081048000	142270128000
(1,2,0,0,1,1,0,0)	33873840	10438428000	75356321040	358102704960
(1,1,3,1,0,0,0,0)	6800713920	4434287457600	48904478663040	588210536833920
(1,1,2,0,0,0,1,0)	0	7607920320	58784806080	255117582720
(1,1,1,1,0,1,0,0)	84324240	24197453280	165692807280	857832655680
(1,1,1,0,2,0,0,0)	166966800	30774744000	207787179600	1111398288000
(1,1,0,2,1,0,0,0)	138378240	21425564160	140903642880	793253260800
(1,1,0,0,1,0,0,1)	0	5765760	28828800	80720640
(1,1,0,0,0,1,1,0)	567840	11444160	46257120	125973120
(1,0,2,1,1,0,0,0)	178017840	64005701760	403020137520	2553954923520
(1,0,1,3,0,0,0,0)	72072000	43312389120	265435410240	1831758888960
(1,0,1,1,0,0,0,1)	0	17297280	86486400	242161920
(1,0,1,0,1,0,1,0)	2162160	45645600	182342160	504504000
(1,0,0,2,0,0,1,0)	1647360	35418240	140849280	392071680
(1,0,0,0,0,0,1,1)	480	2880	7200	13440
(0,8,0,0,0,0,0,0)	31531500	25478650197000	991959264796500	22407816204774000
(0,6,0,1,0,0,0,0)	630630000	7559235684000	189053162298000	3540367414584000
(0,5,2,0,0,0,0,0)	4288284000	16204163976000	373314545604000	5910783999120000
(0,5,0,0,0,1,0,0)	0	53981928000	1036755720000	13695265584000
(0,4,1,0,1,0,0,0)	0	176071896000	2568429864000	33942020112000
(0,4,0,2,0,0,0,0)	-475675200	1253912259600	13778418654000	163679252536800
(0,4,0,0,0,0,0,1)	0	126126000	1891890000	8828820000
(0,3,2,1,0,0,0,0)	3380176800	3693473784000	38931009717600	432879163516800
(0,3,0,1,0,1,0,0)	-3603600	21196375200	144032288400	835444209600
(0,2,4,0,0,0,0,0)	1688406720	2364354632640	24199664448960	289054894128000
(0,2,2,0,0,1,0,0)	-3363360	32590958400	212971318560	1348586279040
(0,2,1,1,1,0,0,0)	190990800	45153108000	291426735600	2252610360000
(0,2,0,3,0,0,0,0)	176576400	26291865600	170151181200	1529310182400
(0,2,0,1,0,0,0,1)	0	21621600	108108000	302702400
(0,2,0,0,0,2,0,0)	2012920	54767440	206835720	617125600
(0,1,3,0,1,0,0,0)	147987840	55973037120	349782713280	3442964004480
(0,1,2,2,0,0,0,0)	191711520	30007897920	192125213280	2349737470080
(0,1,2,0,0,0,0,1)	0	40360320	201801600	565044480
(0,1,1,0,1,1,0,0)	9049040	359879520	1269668400	4137573440
(0,1,0,2,0,1,0,0)	5045040	258017760	879999120	2995312320
(0,1,0,0,0,1,0,1)	7280	43680	109200	203840
(0,0,2,0,2,0,0,0)	4948944	582694112	1853163312	6894119232
(0,0,1,2,1,0,0,0)	2882880	864864000	2655132480	10320710400
(0,0,1,0,1,0,0,1)	32032	192192	480480	896896
(0,0,0,4,0,0,0,0)	64350	332483580	999728730	3989133720
(0,0,0,2,0,0,0,1)	25740	154440	386100	720720
(0,0,0,0,0,0,0,2)	2	4	6	8

Table II. Strong Coupling Expansion of the Second Moment μ_2 through Fifteenth Order

Partition	Linear Chain (LC)	Plane Square (PSQ)	Simple Cubic (SC)	Body-Centered Cubic (BCC)
- 0 -				
(1,0,0,0,0,0,0,0,0)	0	0	0	0
- 1 -				
(2,0,0,0,0,0,0,0,0)	2	4	6	8
- 2 -				
(3,0,0,0,0,0,0,0,0)	16	64	144	256
- 3 -				
(4,0,0,0,0,0,0,0,0)	90	900	3294	8136
(2,1,0,0,0,0,0,0,0)	12	72	180	336
(0,2,0,0,0,0,0,0,0)	2	4	6	8
- 4 -				
(5,0,0,0,0,0,0,0,0)	384	13056	82944	291840
(3,1,0,0,0,0,0,0,0)	192	3072	12096	30720
(1,2,0,0,0,0,0,0,0)	64	256	576	1024
- 5 -				
(6,0,0,0,0,0,0,0,0)	1200	202560	2347920	11847360
(4,1,0,0,0,0,0,0,0)	1440	88560	594000	2155680
(3,0,1,0,0,0,0,0,0)	0	720	3600	10080
(2,2,0,0,0,0,0,0,0)	1170	13140	48870	124200
(1,1,1,0,0,0,0,0,0)	60	360	900	1680
(0,0,2,0,0,0,0,0,0)	2	4	6	8
- 6 -				
(7,0,0,0,0,0,0,0,0)	4320	3412800	74766240	542056320
(5,1,0,0,0,0,0,0,0)	1440	2243520	27384480	144708480
(4,0,1,0,0,0,0,0,0)	0	40320	336960	1313280
(3,2,0,0,0,0,0,0,0)	14880	575040	3659040	13307520
(2,1,1,0,0,0,0,0,0)	1440	26880	108000	276480
(1,3,0,0,0,0,0,0,0)	960	14400	51840	136320

(1,0,2,0,0,0,0,0)	96	384	864	1536
(0,2,1,0,0,0,0,0)	160	640	1440	2560
- 7 -				
(8,0,0,0,0,0,0,0)	44100	62881560	2655431100	27689553360
(6,1,0,0,0,0,0,0)	-80640	54855360	1280059200	9861546240
(5,0,1,0,0,0,0,0)	0	1391040	21833280	126080640
(4,2,0,0,0,0,0,0)	116340	21602280	246996540	1311379440
(4,0,0,1,0,0,0,0)	0	5040	75600	352800
(3,1,1,0,0,0,0,0)	15960	1459920	10117800	38004960
(2,3,0,0,0,0,0,0)	31920	1559040	9802800	37242240
(2,1,0,1,0,0,0,0)	0	10080	50400	141120
(2,0,2,0,0,0,0,0)	2940	36680	139524	359856
(1,2,1,0,0,0,0,0)	6720	82880	309120	806400
(1,0,1,1,0,0,0,0)	112	672	1680	3136
(0,4,0,0,0,0,0,0)	2310	30380	104370	294840
(0,2,0,1,0,0,0,0)	140	840	2100	3920
(0,0,0,2,0,0,0,0)	2	4	6	8
- 8 -				
(9,0,0,0,0,0,0,0)	322560	1264112640	104309130240	1565864294400
(7,1,0,0,0,0,0,0)	-483840	1363944960	62575027200	701510906880
(6,0,1,0,0,0,0,0)	0	40158720	1278789120	11075097600
(5,2,0,0,0,0,0,0)	120960	733420800	15951600000	124944422400
(5,0,0,1,0,0,0,0)	0	322560	8709120	58060800
(4,1,1,0,0,0,0,0)	53760	64942080	818657280	4525086720
(3,3,0,0,0,0,0,0)	470400	108998400	1208511360	6627264000
(3,1,0,1,0,0,0,0)	0	806400	7015680	27740160
(3,0,2,0,0,0,0,0)	43904	2245376	14765184	54602240
(2,2,1,0,0,0,0,0)	170240	8798720	56448000	216903680
(2,0,1,1,0,0,0,0)	3584	68096	274176	702464
(1,4,0,0,0,0,0,0)	116480	4067840	23788800	95083520
(1,2,0,1,0,0,0,0)	4480	125440	524160	1361920
(1,1,2,0,0,0,0,0)	11648	166656	604800	1573376
(1,0,0,2,0,0,0,0)	128	512	1152	2048
(0,3,1,0,0,0,0,0)	4480	107520	362880	1039360
(0,1,1,1,0,0,0,0)	896	3584	8064	14336
- 9 -				
(10,0,0,0,0,0,0,0)	-816480	27569263680	4495820565600	97203332016000
(8,1,0,0,0,0,0,0)	5443200	35322557760	3230986086720	52628686346880
(7,0,1,0,0,0,0,0)	0	1079386560	73869122880	954788446080
(6,2,0,0,0,0,0,0)	-6342840	23882780880	1027201989240	11863433564640
(6,0,0,1,0,0,0,0)	0	10523520	663798240	6885648000
(5,1,1,0,0,0,0,0)	-861840	2521683360	61070662800	503671089600
(5,0,0,0,1,0,0,0)	0	0	1360800	12700800
(4,3,0,0,0,0,0,0)	710640	5821653600	120618726480	973073183040
(4,1,0,1,0,0,0,0)	0	42910560	726939360	4386312000
(4,0,2,0,0,0,0,0)	410760	114402960	1352022840	7433002080
(3,2,1,0,0,0,0,0)	1985760	649837440	7657040160	43856789760
(3,1,0,0,1,0,0,0)	0	151200	2268000	10584000

(Table continued)

Table II. *(Continued)*

(3,0,1,1,0,0,0,0)	52416	5711328	40515552	154808640
(2,4,0,0,0,0,0,0)	3351600	442612800	4551377040	26173183680
(2,2,0,1,0,0,0,0)	65520	12605040	89903520	349322400
(2,1,2,0,0,0,0,0)	355824	21141792	136615248	533937600
(2,0,1,0,1,0,0,0)	0	30240	151200	423360
(2,0,0,2,0,0,0,0)	6138	83556	323838	842760
(1,3,1,0,0,0,0,0)	569520	34191360	207612720	871799040
(1,2,0,0,1,0,0,0)	0	75600	378000	1058400
(1,1,1,1,0,0,0,0)	53928	712656	2689848	7086240
(1,0,0,1,1,0,0,0)	180	1080	2700	5040
(0,2,2,0,0,0,0,0)	53424	852768	2936304	8668800
(0,1,1,0,1,0,0,0)	840	5040	12600	23520
(0,0,0,0,2,0,0,0)	2	4	6	8

– 10 –

(11,0,0,0,0,0,0,0)	-18144000	648321408000	211145941036800	6576908186419200
(9,1,0,0,0,0,0,0)	29030400	962952883200	176930203564800	4183098750028800
(8,0,1,0,0,0,0,0)	0	29393280000	4373643859200	83731439232000
(7,2,0,0,0,0,0,0)	53222400	774464544000	67476683232000	1145876738227200
(7,0,0,1,0,0,0,0)	0	250387200	45031593600	731195942400
(6,1,1,0,0,0,0,0)	-6652800	89360409600	4398829545600	54405867801600
(6,0,0,0,1,0,0,0)	0	0	179625600	2467584000
(5,3,0,0,0,0,0,0)	-109468800	268696915200	10926042825600	130711362163200
(5,1,0,1,0,0,0,0)	0	1854316800	64207987200	588569587200
(5,0,2,0,0,0,0,0)	2298240	4975488000	112057706880	920602851840
(4,2,1,0,0,0,0,0)	-4233600	38957990400	871058966400	7408964505600
(4,1,0,0,1,0,0,0)	0	12096000	348364800	2370816000
(4,0,1,1,0,0,0,0)	282240	336510720	4414435200	24952757760
(3,4,0,0,0,0,0,0)	51559200	36058276800	684945525600	5712234998400
(3,2,0,1,0,0,0,0)	352800	931089600	12381012000	72602812800
(3,1,2,0,0,0,0,0)	4515840	1923909120	22269461760	129615897600
(3,0,1,0,1,0,0,0)	0	3064320	26853120	106444800
(3,0,0,2,0,0,0,0)	106560	7009920	47329920	176302080
(2,3,1,0,0,0,0,0)	17640000	4257590400	46007438400	278141472000
(2,2,0,0,1,0,0,0)	0	8668800	80740800	326592000
(2,1,1,1,0,0,0,0)	1431360	95840640	628568640	2434440960
(2,0,3,0,0,0,0,0)	201600	10725120	69552000	276917760
(2,0,0,1,1,0,0,0)	7200	138240	557280	1428480
(1,5,0,0,0,0,0,0)	3427200	692294400	6619536000	41012697600
(1,3,0,1,0,0,0,0)	201600	34675200	228009600	972518400
(1,2,2,0,0,0,0,0)	3087840	154096320	910012320	3945244800
(1,1,1,0,1,0,0,0)	33600	994560	4173120	10859520
(1,1,0,2,0,0,0,0)	46080	633600	2315520	5967360
(1,0,2,1,0,0,0,0)	40320	584640	2116800	5523840
(1,0,0,0,2,0,0,0)	160	640	1440	2560
(0,4,1,0,0,0,0,0)	1142400	42470400	242524800	1116595200
(0,3,0,0,1,0,0,0)	0	268800	1209600	3225600
(0,2,1,1,0,0,0,0)	80640	1612800	5564160	15482880
(0,1,3,0,0,0,0,0)	20160	1048320	3326400	10321920
(0,1,0,1,1,0,0,0)	1920	7680	17280	30720
(0,0,2,0,1,0,0,0)	2016	8064	18144	32256

- 11 -

(12,0,0,0,0,0,0,0)	384199200	16360978233600	10736614099965600	481897820225606400
(10,1,0,0,0,0,0,0)	-1766318400	27664617657600	10281061529323200	352461407405184000
(9,0,1,0,0,0,0,0)	0	836177126400	268896888691200	7585473728448000
(8,2,0,0,0,0,0,0)	2799165600	25654297838400	4573613240935200	113877668222841600
(8,0,0,1,0,0,0,0)	0	5019537600	2982623212800	74756921500800
(7,1,1,0,0,0,0,0)	133056000	3059955360000	316787987577600	5859677120774400
(7,0,0,0,1,0,0,0)	0	0	14639486400	336378873600
(6,3,0,0,0,0,0,0)	-1570060800	11458104134400	948968531280000	16827326933155200
(6,1,0,1,0,0,0,0)	0	68866459200	5230253397600	73509421708800
(6,0,2,0,0,0,0,0)	5544000	199258456320	8891779881600	109806171995520
(6,0,0,0,0,1,0,0)	0	0	14968800	419126400
(5,2,1,0,0,0,0,0)	-358696800	1974462336000	88063045408800	1121454135648000
(5,1,0,0,1,0,0,0)	0	488980800	37901001600	417030768000
(5,0,1,1,0,0,0,0)	-2772000	16705624320	426032339040	3660594981120
(4,4,0,0,0,0,0,0)	24047100	2403318040200	87898800874500	1082374079094000
(4,2,0,1,0,0,0,0)	-3465000	55519002000	1473900813000	13280212360800
(4,1,2,0,0,0,0,0)	-1774080	129286080000	2869011351360	25173748575360
(4,1,0,0,0,1,0,0)	0	0	74844000	698544000
(4,0,1,0,1,0,0,0)	0	224199360	3953759040	24377189760
(4,0,0,2,0,0,0,0)	1047420	498417480	6144080580	34560353520
(3,3,1,0,0,0,0,0)	139154400	388578960000	7879112287200	70300244044800
(3,2,0,0,1,0,0,0)	0	716839200	13177533600	83096798400
(3,1,1,1,0,0,0,0)	20180160	8709845760	104731704000	616122904320
(3,0,3,0,0,0,0,0)	0	1199056320	15620109120	96936174720
(3,0,1,0,0,1,0,0)	0	665280	.. 9979200	46569600
(3,0,0,1,1,0,0,0)	130680	17178480	124704360	482470560
(2,5,0,0,0,0,0,0)	172557000	120195860400	2160517174200	19222620832800
(2,3,0,1,0,0,0,0)	2910600	5963680800	75202835400	480991896000
(2,2,2,0,0,0,0,0)	93970800	22169790720	236877102000	1519338522240
(2,2,0,0,0,1,0,0)	0	2494800	37422000	174636000
(2,1,1,0,1,0,0,0)	609840	143146080	1035674640	4074396480
(2,1,0,2,0,0,0,0)	1591920	105407280	699922080	2761846560
(2,0,2,1,0,0,0,0)	1413720	91753200	596340360	2377821600
(2,0,0,1,0,1,0,0)	0	71280	356400	997920
(2,0,0,0,2,0,0,0)	11484	167112	656964	1718640
(1,4,1,0,0,0,0,0)	82882800	12135261600	119909235600	777010449600
(1,3,0,0,1,0,0,0)	0	52668000	384199200	1540123200
(1,2,1,1,0,0,0,0)	6264720	557172000	3486233520	15289686720
(1,1,3,0,0,0,0,0)	5932080	404822880	2379540240	11077577280
(1,1,1,0,0,1,0,0)	0	665280	3326400	9313920
(1,1,0,1,1,0,0,0)	162360	2288880	8755560	23205600
(1,0,2,0,1,0,0,0)	155232	2128896	8049888	21377664
(1,0,0,0,1,1,0,0)	264	1584	3960	7392
(0,6,0,0,0,0,0,0)	5313000	1313466000	11684395800	84866628000
(0,4,0,1,0,0,0,0)	1386000	149965200	886485600	4328200800
(0,3,2,0,0,0,0,0)	2079000	117255600	674843400	3331389600
(0,3,0,0,0,1,0,0)	0	277200	1386000	3880800
(0,2,0,2,0,0,0,0)	280170	4765860	16830990	49353480
(0,1,2,1,0,0,0,0)	388080	7318080	24781680	76507200
(0,1,0,1,0,1,0,0)	1980	11880	29700	55440
(0,0,4,0,0,0,0,0)	47124	2123352	6627852	24049872
(0,0,2,0,0,1,0,0)	1848	11088	27720	51744
(0,0,0,0,0,2,0,0)	2	4	6	8

(*Table continued*)

Table II. *(Continued)*

- 12 -

(13,0,0,0,0,0,0,0)	5748019200	441390394368000	5878700818887334400	38027308780829491200
(11,1,0,0,0,0,0,0)	-14849049600	836636648601600	633457567556352000	31464224997288652800
(10,0,1,0,0,0,0,0)	0	25094893824000	17289689687424000	716023423019520000
(9,2,0,0,0,0,0,0)	3672345600	877764045312000	321877584909619200	11724397683457536000
(9,0,0,1,0,0,0,0)	0	99632332800	201478131993600	7669231389388800
(8,1,1,0,0,0,0,0)	958003200	105370133299200	23236893559756800	639703916070912000
(8,0,0,0,1,0,0,0)	0	0	1053324518400	40755372134400
(7,3,0,0,0,0,0,0)	17403724800	475170385536000	81619577743334400	2139164832894566400
(7,1,0,1,0,0,0,0)	0	2362675392000	412270210598400	8869902068966400
(7,0,2,0,0,0,0,0)	149022720	7531139911680	692918828743680	12866924586700800
(7,0,0,0,0,1,0,0)	0	0	2155507200	91968307200
(6,2,1,0,0,0,0,0)	1277337600	90001526630400	8384445922406400	160313433512601600
(6,1,0,0,1,0,0,0)	0	14609548800	3528325785600	61739314560000
(6,0,1,1,0,0,0,0)	-26611200	725054077440	37781251200000	493378087910400
(5,4,0,0,0,0,0,0)	-14396659200	137868688742400	10216506813004800	187239659878656000
(5,2,0,1,0,0,0,0)	-33264000	2820827116800	158509353340800	2188229582246400
(5,1,2,0,0,0,0,0)	-968647680	7368918036480	326480305536000	4316194131701760
(5,1,0,0,0,1,0,0)	0	0	12214540800	172440576000
(5,0,1,0,1,0,0,0)	0	12214540800	447786662400	4225549870080
(5,0,0,2,0,0,0,0)	6747840	27628318080	662008438080	5619639548160
(4,3,1,0,0,0,0,0)	-5229100800	28408387392000	1125861766444800	14874861026995200
(4,2,0,0,1,0,0,0)	0	43948396800	1700176262400	16655231577600
(4,1,1,1,0,0,0,0)	66528000	619285201920	14423624328960	129066683074560
(4,0,3,0,0,0,0,0)	-79833600	109137853440	2657931978240	24900568197120
(4,0,1,0,0,1,0,0)	0	63866880	1868106240	12773376000
(4,0,0,1,1,0,0,0)	950400	1308890880	17886718080	102543598080
(3,5,0,0,0,0,0,0)	2162160000	13706098560000	461491516886400	5958030798720000
(3,3,0,1,0,0,0,0)	-93139200	628942406400	15043644000000	142708387852800
(3,2,2,0,0,0,0,0)	1108356480	2188230105600	44038077874560	414295679508480
(3,2,0,0,0,1,0,0)	0	239500800	7664025600	53754624000
(3,1,1,0,1,0,0,0)	4435200	13032391680	179713416960	1072970680320
(3,1,0,2,0,0,0,0)	20813760	11928090240	143528552640	850214534400
(3,0,2,1,0,0,0,0)	14636160	11919156480	141884265600	857279808000
(3,0,0,1,0,1,0,0)	0	8743680	76982400	305648640
(3,0,0,0,2,0,0,0)	217536	17888640	123114816	460255488
(2,4,1,0,0,0,0,0)	2621203200	2081971584000	38248317676800	361727425382400
(2,3,0,0,1,0,0,0)	0	7460006400	109571616000	702695347200
(2,2,1,1,0,0,0,0)	198696960	91611717120	1032943000320	6737175244800
(2,1,3,0,0,0,0,0)	185391360	54920194560	584065278720	4024816266240
(2,1,1,0,0,1,0,0)	0	94026240	886152960	3597834240
(2,1,0,1,1,0,0,0)	4815360	416528640	2797027200	10838615040
(2,0,2,0,1,0,0,0)	4435200	330156288	2168812800	8458458624
(2,0,1,2,0,0,0,0)	2661120	164229120	1080034560	4300369920
(2,0,0,0,1,1,0,0)	12672	244992	988416	2534400
(1,6,0,0,0,0,0,0)	565488000	292967136000	4907311516800	48553491571200
(1,4,0,1,0,0,0,0)	62092800	29591654400	327038342400	2307386188800
(1,3,2,0,0,0,0,0)	392071680	87533994240	861518972160	6011407987200
(1,3,0,0,0,1,0,0)	0	53222400	558835200	2341785600
(1,2,1,0,1,0,0,0)	2217600	751322880	5030847360	21262348800
(1,2,0,2,0,0,0,0)	15903360	1018195200	6187674240	26540997120
(1,1,2,1,0,0,0,0)	30602880	2096962560	12444727680	58090475520
(1,1,0,1,0,1,0,0)	95040	2914560	12260160	31933440
(1,1,0,0,2,0,0,0)	137280	1837440	6747840	17276160
(1,0,4,0,0,0,0,0)	4169088	460551168	2530459008	13539068928
(1,0,2,0,0,1,0,0)	88704	2838528	11975040	31223808
(1,0,1,1,1,0,0,0)	392832	5601024	20338560	52867584
(1,0,0,0,0,2,0,0)	192	768	1728	3072
(0,5,1,0,0,0,0,0)	37699200	15554246400	136122940800	998212723200
(0,4,0,0,1,0,0,0)	0	79833600	518918400	2483712000
(0,3,1,1,0,0,0,0)	17740800	1332334080	7605480960	41006085120
(0,2,3,0,0,0,0,0)	24097920	931096320	5275670400	28318456320

(0,2,1,0,0,1,0,0)	0	4730880	21288960	56770560
(0,2,0,1,1,0,0,0)	475200	8236800	28987200	78566400
(0,1,2,0,1,0,0,0)	295680	6209280	21288960	59727360
(0,1,1,2,0,0,0,0)	506880	17614080	57404160	172085760
(0,1,0,0,1,1,0,0)	3520	14080	31680	56320
(0,0,3,1,0,0,0,0)	88704	12595968	38586240	125250048
(0,0,1,1,0,1,0,0)	12672	50688	114048	202752

- 13 -

(14,0,0,0,0,0,0,0)	-96129633600	12685383984873600	34490407873708987200	3215781812947338835200
(12,1,0,0,0,0,0,0)	484929244800	2661045261142400	41309801005487414400	2970947970096756249600
(11,0,1,0,0,0,0,0)	0	790146669312000	1165592302082208000	70658256635942976000
(10,2,0,0,0,0,0,0)	-865231567200	31137361165310400	23595870912581167200	1254826336906721779200
(10,0,0,1,0,0,0,0)	0	1857208953600	14040707516035200	803208913374067200
(9,1,1,0,0,0,0,0)	-50724273600	3734827746249600	1753637347117656000	71537980382899430400
(9,0,0,0,1,0,0,0)	0	0	74539774108800	4703664226060800
(8,3,0,0,0,0,0,0)	612323712000	19715006274163200	7070599854279561600	273047547748719744000
(8,1,0,1,0,0,0,0)	0	80295098083200	32493732522451200	1063194077648102400
(8,0,2,0,0,0,0,0)	5695129440	281634823229760	54421304951613600	1514223689493715200
(8,0,0,0,0,1,0,0)	0	0	168129561600	13532872953600
(7,2,1,0,0,0,0,0)	152562009600	3889010424499200	778410150972854400	22285285349892364800
(7,1,0,0,1,0,0,0)	0	382183401600	303220107590400	8449853950137600
(7,0,1,1,0,0,0,0)	864864000	29448169470720	3241904252146560	64394738528440320
(7,0,0,0,0,0,1,0)	0	0	0	10897286400
(6,4,0,0,0,0,0,0)	-69989119200	7262926791120000	1125538894892412000	30708442430640374400
(6,2,0,1,0,0,0,0)	1081080000	128388347287200	15972548286895200	338119383437035200
(6,1,2,0,0,0,0,0)	-10724313600	370901250720000	34234160810129280	683480718429949440
(6,1,0,0,0,1,0,0)	0	0	1292301410400	32354302780800
(6,0,1,0,1,0,0,0)	0	541612431360	45151972145280	667754956028160
(6,0,0,2,0,0,0,0)	35135100	1362384583560	65944962703620	857207568737520
(5,3,1,0,0,0,0,0)	-122291769600	1769829797736000	143463293985211200	2835522262766793600
(5,2,0,0,1,0,0,0)	0	2107219514400	193610011795200	3014993439057600
(5,1,1,1,0,0,0,0)	-2090808720	36768469177440	1742869584696240	23917517176512960
(5,1,0,0,0,0,1,0)	0	0	1362160800	38140502400
(5,0,3,0,0,0,0,0)	0	7041238364160	354424743953280	5062148278387200
(5,0,1,0,0,1,0,0)	0	3078915840	268241893920	3041640201600
(5,0,0,1,1,0,0,0)	-6640920	83425708080	2277114479640	20211182877600
(4,5,0,0,0,0,0,0)	-92324232000	1171196558532000	78061950456890400	1489969914093460800
(4,3,0,1,0,0,0,0)	-3059456400	48395411064000	2374781052243600	34170275537798400
(4,2,2,0,0,0,0,0)	-11728276560	174449106992160	6918350718399120	97197327076449600
(4,2,0,0,0,1,0,0)	0	11545934400	1187090704800	13886256384000
(4,1,1,0,1,0,0,0)	-30990960	950463914400	26332685018640	245350653387840
(4,1,0,2,0,0,0,0)	-60386040	978666202800	22702534054200	206134216722720
(4,0,2,1,0,0,0,0)	-67026960	977322265920	23375412300240	219795914257920
(4,0,1,0,0,0,1,0)	0	0	454053600	4237833600
(4,0,0,1,0,1,0,0)	0	858377520	15757513200	98864457120
(4,0,0,0,2,0,0,0)	2016300	1720149288	22018464468	125997169584
(3,4,1,0,0,0,0,0)	15192777600	242071491261600	8603576239658400	119988294642216000
(3,3,0,0,1,0,0,0)	0	736258723200	22797425851200	230224634640000
(3,2,1,1,0,0,0,0)	1785944160	10235602016640	223831356708960	2209313198252160
(3,2,0,0,0,0,1,0)	0	0	2270268000	21189168000
(3,1,3,0,0,0,0,0)	589548960	6055587457920	122458978081440	1259025791063040
(3,1,1,0,0,1,0,0)	0	10362512160	196319803680	1258835497920
(3,1,0,1,1,0,0,0)	75984480	49393309680	617694208560	3689834148000
(3,0,2,0,1,0,0,0)	74234160	36643134528	434992429872	2591672219136
(3,0,1,2,0,0,0,0)	0	21926464560	293349204720	1854785481120
(3,0,0,1,0,0,1,0)	0	2162160	32432400	151351200
(3,0,0,0,1,1,0,0)	274560	43661904	323479728	1262358240
(2,6,0,0,0,0,0,0)	26959432500	55730248774200	1709467374314700	23847629806400400

(Table continued)

Table II. *(Continued)*

(2,4,0,1,0,0,0,0)	762161400	4822542925200	100019159440200	1026416811420000
(2,3,2,0,0,0,0,0)	14856201360	16441964098560	306014342634000	3113775826761600
(2,3,0,0,0,1,0,0)	0	7304497200	145697151600	958190032800
(2,2,1,0,1,0,0,0)	39639600	141533552160	1824666283440	11998653226560
(2,2,0,2,0,0,0,0)	536112720	168328892160	1839016848240	12072571917120
(2,1,2,1,0,0,0,0)	808647840	394065191520	4428086382720	31759700132160
(2,1,1,0,0,0,1,0)	0	30270240	454053600	2118916800
(2,1,0,1,0,1,0,0)	2059200	587592720	4318451280	17134397280
(2,1,0,0,2,0,0,0)	5357352	400857600	2732918760	10860440448
(2,0,4,0,0,0,0,0)	325284960	84974233344	846533207520	6462117742080
(2,0,2,0,0,1,0,0)	1921920	544576032	3968284320	15775311552
(2,0,1,1,1,0,0,0)	16535376	1118742768	7367529312	29577813408
(2,0,0,0,1,0,1,0)	0	144144	720720	2018016
(2,0,0,0,0,2,0,0)	19890	304148	1208454	3170856
(1,5,1,0,0,0,0,0)	6675669000	6487615134000	112177774308600	1197666413143200
(1,4,0,0,1,0,0,0)	0	37423386000	525048123600	3978713138400
(1,3,1,1,0,0,0,0)	1584863280	523743620400	5414394585600	40678251213600
(1,3,0,0,0,0,1,0)	0	25225200	378378000	1765764000
(1,2,3,0,0,0,0,0)	1839037200	325688563200	3158195760720	24115996544640
(1,2,1,0,0,1,0,0)	0	1223542320	8986657680	36319963680
(1,2,0,1,1,0,0,0)	31351320	3503522880	22740208920	99931328640
(1,1,2,0,1,0,0,0)	21477456	2761318560	17362288944	79872496704
(1,1,1,2,0,0,0,0)	89214840	7653428640	46012463640	219166833600
(1,1,0,1,0,0,1,0)	0	2162160	10810800	30270240
(1,1,0,0,1,1,0,0)	410124	6100952	23620740	62812464
(1,0,3,1,0,0,0,0)	42618576	4994205216	28485593136	154026320448
(1,0,2,0,0,0,1,0)	0	2018016	10090080	28252224
(1,0,1,1,0,1,0,0)	1257828	18028296	68638284	183275664
(1,0,0,0,0,1,1,0)	364	2184	5460	10192
(0,4,2,0,0,0,0,0)	1044743700	248703294840	2238409112940	17898858017040
(0,3,1,0,1,0,0,0)	18258240	4665220560	28375467120	144878653920
(0,2,2,1,0,0,0,0)	106786680	8538610080	48273465240	282590468160
(0,2,1,0,0,0,1,0)	0	5045040	25225200	70630560
(0,2,0,0,2,0,0,0)	1135134	20112092	72786714	211050840
(0,1,1,1,1,0,0,0)	4851132	95584632	327345876	1009138416
(0,1,0,0,1,0,1,0)	4004	24024	60060	112112
(0,0,2,2,0,0,0,0)	1818102	88845900	278463042	1012285560
(0,0,1,1,0,0,1,0)	12012	72072	180180	336336
(0,0,0,0,0,0,2,0)	2	4	6	8

– 14 –

(15,0,0,0,0,0,0,0)	−1754463110400	386947100533593600	215905607605199191680 0	290180985885633338496000
(13,1,0,0,0,0,0,0)	5132621894400	88958229986545920 0	28457503436468538432 00	29618729975302797288960 0
(12,0,1,0,0,0,0,0)	0	25977169266048000	824732978288034816 00	7301251371800613427200
(11,2,0,0,0,0,0,0)	−1928819692800	114633117245687040 0	180443229263331914880 0	1398685918481219047680 00
(11,0,0,1,0,0,0,0)	0	16716437337600	10169276509199808 00	868159016792331264 00
(10,1,1,0,0,0,0,0)	−515804889600	137073609261388800	13689281486730769920 0	824564979630035804160 0
(10,0,0,0,1,0,0,0)	0	0	5432635086278400	541922096261145600
(9,3,0,0,0,0,0,0)	−7384727750400	830529447177830400	623479734572831136000	35382145250784210278400
(9,1,0,1,0,0,0,0)	0	2834035479475200	2606864021630668800	1286653934066941440 00
(9,0,2,0,0,0,0,0)	41167526400	1070970051191040 0	4358428562821132800	180820928803971456000
(9,0,0,0,0,1,0,0)	0	0	11147923987200	1776214094054400
(8,2,1,0,0,0,0,0)	259113254400	166596087155097600	72090757174813977600	3076881729978760704000
(8,1,0,0,1,0,0,0)	0	10330627507200	25410935367417600	11151873599682816 00
(8,0,1,1,0,0,0,0)	7749181440	1167853465658880	2750549188907520 00	8267910176155069440
(8,0,0,0,0,0,1,0)	0	0	0	2615348736000
(7,4,0,0,0,0,0,0)	9220315104000	367360470520243200	120941551885656902400	4899911374899033292800

(7,2,0,1,0,0,0,0)	9686476800	5491774659571200	1557906646978281600	50404812044261875200
(7,1,2,0,0,0,0,0)	147234447360	17332338888184320	3447755647592256000	103881246925343247360
(7,1,0,0,0,1,0,0)	0	0	118715038041600	5131721052057600
(7,0,1,0,1,0,0,0)	0	21762607426560	4241879157035520	97489454700718080
(7,0,0,2,0,0,0,0)	380540160	61367082416640	6199108801966080	122901936825507840
(6,3,1,0,0,0,0,0)	1433598566400	98450175452428800	17052041876892422400	505924880558159462400
(6,2,0,0,1,0,0,0)	0	87141966912000	20356623189302400	501144530465779200
(6,1,1,1,0,0,0,0)	12592419840	1918586898708480	194396065132389120	4087124347033728000
(6,1,0,0,0,0,1,0)	0	0	228843014400	10112681779200
(6,0,3,0,0,0,0,0)	33902668800	396787633482240	42636561488801280	927504446315166720
(6,0,1,0,0,1,0,0)	0	110425835520	30781928136960	560060464803840
(6,0,0,1,1,0,0,0)	-77837760	4382681022720	251139822373440	3433932133632000
(5,5,0,0,0,0,0,0)	-3741401664000	84015549973555200	11594536998143299200	329735170222813555200
(5,3,0,1,0,0,0,0)	2421619200	3086335508256000	326439856046304000	7134669283745203200
(5,2,2,0,0,0,0,0)	-414904089600	11757067928686080	960749585930534400	20220572117143157760
(5,2,0,0,0,1,0,0)	0	414096883200	147787181942400	2807431570944000
(5,1,1,0,1,0,0,0)	-363242880	56269228055040	3342264938812800	48543669026012160
(5,1,0,2,0,0,0,0)	-5336210880	65151640794240	3072038830660800	42800418103322880
(5,0,2,1,0,0,0,0)	-1271350080	67885008871680	3352733780235840	48733731746219520
(5,0,1,0,0,0,1,0)	0	0	87178291200	1239869030400
(5,0,0,1,0,1,0,0)	0	57548050560	2248058292480	21692207496960
(5,0,0,0,2,0,0,0)	15567552	117586909440	3001460349888	26103394437120
(4,4,1,0,0,0,0,0)	-1364178816000	22106585518617600	1576370889184291200	32599292601856780800
(4,3,0,0,1,0,0,0)	0	55166906995200	3785161533753600	59411424264192000
(4,2,1,1,0,0,0,0)	-65464439040	884580362265600	38725206472592640	574886415389045760
(4,2,0,0,0,0,1,0)	0	0	452237385600	6635236608000
(4,1,3,0,0,0,0,0)	-68572183680	517144529341440	20767460088735360	316411259908423680
(4,1,1,0,0,1,0,0)	0	755948793600	30972751729920	311367114938880
(4,1,0,1,1,0,0,0)	345945600	4260683306880	105463211293440	969647843324160
(4,0,2,0,1,0,0,0)	698233536	3030817870080	71290398763584	656603803215360
(4,0,1,2,0,0,0,0)	-1452971520	2504438576640	63757272458880	618261107143680
(4,0,0,1,0,0,1,0)	0	242161920	7161073920	49124275200
(4,0,0,0,1,1,0,0)	2498496	4103299200	58054860864	335840144640
(3,6,0,0,0,0,0,0)	404006803200	7327018950451200	435356533637625600	8772840838335974400
(3,4,0,1,0,0,0,0)	-24014390400	557182087862400	22994258692204800	348251821722604800
(3,3,2,0,0,0,0,0)	44638513920	2130151417954560	76307189889490560	1143342096359823360
(3,3,0,0,0,1,0,0)	0	621347126400	28400749977600	303256547193600
(3,2,1,0,1,0,0,0)	-1291530240	17375117760000	440374115301120	4446381346767360
(3,2,0,2,0,0,0,0)	7534406880	19387549621440	406709314371360	4039527970558080
(3,1,2,1,0,0,0,0)	4237833600	47548573712640	1020477958980480	10935661638021120
(3,1,1,0,0,0,1,0)	0	3390266880	111152321280	784604620800
(3,1,0,1,0,1,0,0)	18738720	66787680960	959579580960	5789693669760
(3,1,0,0,2,0,0,0)	73032960	57254765568	718515173376	4293384775680
(3,0,4,0,0,0,0,0)	4221689472	8958538068480	170744390264448	1929257091041280
(3,0,2,0,0,1,0,0)	17489472	58839965184	824993193024	4991086324224
(3,0,1,1,1,0,0,0)	187963776	173985267456	2104124166144	12791518451712
(3,0,0,3,0,0,0,0)	0	10793502720	144051747840	906169904640
(3,0,0,0,1,0,1,0)	0	20756736	183351168	728792064
(3,0,0,0,0,2,0,0)	396032	39603200	276389568	1035157760
(2,5,1,0,0,0,0,0)	264057393600	1312922824012800	42367466259518400	640970934725721600
(2,4,0,0,1,0,0,0)	0	6555121372800	175712689152000	1947661100985600
(2,3,1,1,0,0,0,0)	50329319040	97577168008320	1922434004050560	20989689297857280
(2,3,0,0,0,0,1,0)	0	2825222400	108972864000	799134336000
(2,2,3,0,0,0,0,0)	62074172160	73668911796480	1342611431441280	14948583964554240
(2,2,1,0,0,1,0,0)	0	203550547200	3103264644480	20126184798720
(2,2,0,1,1,0,0,0)	978737760	689662733760	8126577819360	53469530674560
(2,1,2,0,1,0,0,0)	730521792	715292497920	8308038682944	57652887452160
(2,1,1,2,0,0,0,0)	2656573920	1303618155840	14373334815840	103996557624960
(2,1,0,1,0,0,1,0)	0	363242880	3450807360	14045391360
(2,1,0,0,1,1,0,0)	13037024	1425359936	9751918176	37710248576

(Table continued)

Table II. *(Continued)*

(2,0,3,1,0,0,0,0)	1778544768	809041449216	8504484468480	69824914199040
(2,0,2,0,0,0,1,0)	0	344408064	3293402112	13431914496
(2,0,1,1,0,1,0,0)	41032992	3598795200	23957021088	93614801280
(2,0,1,0,2,0,0,0)	11531520	890233344	5967561600	23704192512
(2,0,0,2,1,0,0,0)	12108096	711494784	4658157504	18549603072
(2,0,0,0,0,1,1,0)	20384	396032	1598688	4100096
(1,7,0,0,0,0,0,0)	28857628800	100577513836800	2813015219414400	43623744080716800
(1,5,0,1,0,0,0,0)	2018016000	12956873529600	244231193606400	2824618609612800
(1,4,2,0,0,0,0,0)	96319903680	75114739539840	1281229136066880	14556868089603840
(1,4,0,0,0,1,0,0)	0	28723094400	519033715200	3878492217600
(1,3,1,0,1,0,0,0)	975374400	1083580417920	12530446568640	97154205308160
(1,3,0,2,0,0,0,0)	1870989120	708144877440	7152814468800	55152527189760
(1,2,2,1,0,0,0,0)	9659569920	3533976526080	35664033525120	299119910369280
(1,2,1,0,0,0,1,0)	0	1183902720	12592419840	52952739840
(1,2,0,1,0,1,0,0)	8648640	4543418880	31011140160	128768640000
(1,2,0,0,2,0,0,0)	66370304	5420455040	34077947904	143792160512
(1,1,4,0,0,0,0,0)	3866518656	745137609216	6960920174208	58274296464384
(1,1,2,0,0,1,0,0)	8072064	6255849600	41966660736	177666128640
(1,1,1,1,1,0,0,0)	386402016	30377290944	182483132832	836532977280
(1,1,0,3,0,0,0,0)	40360320	3770807040	23437814400	117529251840
(1,1,0,0,1,0,1,0)	224224	7047040	29693664	77389312
(1,1,0,0,0,2,0,0)	337792	4437888	16353792	41676544
(1,0,3,0,1,0,0,0)	26906880	3853065216	23812588800	123330375168
(1,0,2,2,0,0,0,0)	161249088	25456214784	141832506816	778715857920
(1,0,1,1,0,0,1,0)	672672	22678656	95999904	250618368
(1,0,1,0,1,1,0,0)	1473472	20564544	74954880	193857664
(1,0,0,2,0,1,0,0)	1153152	16528512	59963904	156059904
(1,0,0,0,0,0,2,0)	224	896	2016	3584
(0,6,1,0,0,0,0,0)	11805393600	7231022198400	120895605230400	1488953417952000
(0,5,0,0,1,0,0,0)	0	65989123200	883487404800	8000895302400
(0,4,1,1,0,0,0,0)	2280358080	1000922482560	9279342072000	77123809002240
(0,4,0,0,0,0,1,0)	0	134534400	1816214400	8072064000
(0,3,3,0,0,0,0,0)	2482159680	1098715537920	9382381968960	81098251153920
(0,3,1,0,0,1,0,0)	0	2744501760	18162144000	83949465600
(0,3,0,1,1,0,0,0)	70150080	10751220480	63011108160	328210122240
(0,2,2,0,1,0,0,0)	103591488	20376580224	116193325248	705449961216
(0,2,1,2,0,0,0,0)	404564160	23499315840	133682028480	816366270720
(0,2,0,1,0,0,1,0)	0	19219200	86486400	230630400
(0,2,0,0,1,1,0,0)	1985984	31519488	112432320	299307008
(0,1,3,1,0,0,0,0)	429164736	24931915008	136978890048	993692603904
(0,1,2,0,0,0,1,0)	0	37669632	169513344	452035584
(0,1,1,1,0,1,0,0)	3651648	71495424	247351104	685741056
(0,1,1,0,2,0,0,0)	4196192	106089984	356035680	1027330304
(0,1,0,2,1,0,0,0)	1921920	74186112	239855616	726485760
(0,1,0,0,0,1,1,0)	5824	23296	52416	93184
(0,0,2,1,1,0,0,0)	2978976	222173952	693332640	2197907712
(0,0,1,3,0,0,0,0)	384384	168744576	509693184	1684370688
(0,0,1,0,1,0,1,0)	32032	128128	288288	512512
(0,0,0,2,0,0,1,0)	27456	109824	247104	439296

- 15 -

(16,0,0,0,0,0,0,0)	38310772500000	1248640230039602400 0	14365092490743792798000 0	2783289906775882601697600
(14,1,0,0,0,0,0,0)	-210780762192000	3121932975707145600 0	20664016205042439028800 0	3111194474432149635129600
(13,0,1,0,0,0,0,0)	0	891469731664512000	612118342781455910400 0	789980721094405032192000
(12,2,0,0,0,0,0,0)	437178697956000	4384072162063994400 0	14398262224045637137200 0	1624533452571185250494400
(12,0,0,1,0,0,0,0)	0	-1152388036800000	767493100164510720 00	97265383890635600640 00
(11,1,1,0,0,0,0,0)	19083872808000	5207563214053200000	1108702107423508197600 0	983091862588798426464000

(11,0,0,0,1,0,0,0)	0	0	4090918685293 44000	6340321026245 6064000
(10,3,0,0,0,0,0,0)	-415540773648000	3573135928222 6608000	5631122190874 3401600000	4684932788007 299379552000
(10,1,0,1,0,0,0,0)	0	1043498809473 12000	2147785769141 81280000	1588785022259 7277 12000
(10,0,2,0,0,0,0,0)	-1656841586400	4195219147046 92800	3589165931200 85800800	2211001557584 4519523200
(10,0,0,0,0,1,0,0)	0	0	757879025904000	2216419785740 16000
(9,2,1,0,0,0,0,0)	-67408797456000	7236456193158 192000	6745004972938 552320000	4273648350019 15146336000
(9,1,0,0,1,0,0,0)	0	286625875536000	2139637315063 248000	1458207348096 05856000
(9,0,1,1,0,0,0,0)	-388669881600	4681718881519 6800	2357375017679 9222400	1066162218518 920723200
(9,0,0,0,0,0,1,0)	0	0	0	388379287296000
(8,4,0,0,0,0,0,0)	161527297932000	1838615413144 9104000	1290183847075 3492260000	7736860881455 60221632000
(8,2,0,1,0,0,0,0)	-485837352000	2323916884729 44000	1506190672147 28112000	7401322274237 027136000
(8,1,2,0,0,0,0,0)	4106460758400	7867683042579 36000	3419930685914 97523200	1548362107340 0588908800
(8,1,0,0,0,1,0,0)	0	0	1022744382660 0000	7475565168751 68000
(8,0,1,0,1,0,0,0)	0	860709452803200	3883824638549 85600	1379811071214 5760000
(8,0,0,2,0,0,0,0)	16086470400	266576678287920 0	5728186755915 62400	1727046722754 7934400
(8,0,0,0,0,0,0,1)	0	0	0	163459296000
(7,3,1,0,0,0,0,0)	72603170640000	5156200925091 216000	1953983434912 773264000	8689648895556 4255968000
(7,2,0,0,1,0,0,0)	0	3454212762000 000	2051632246744 080000	7950544175804 6304000
(7,1,1,1,0,0,0,0)	1560128169600	9316239072940 8000	2069871264652 0708800	6648843136710 43065600
(7,1,0,0,0,0,1,0)	0	0	20718465768000	1924733210400 000
(7,0,3,0,0,0,0,0)	0	1978753783406 4000	4756123019439 792000	1574323274721 63744000
(7,0,1,0,0,1,0,0)	0	3523455936000	3145014065793 600	9360668399322 2400
(7,0,0,1,1,0,0,0)	3891888000	207712267963200	2605350436809 1200	5532567014022 04800
(6,5,0,0,0,0,0,0)	2892321432000	5406548399284 032000	1595338044576 684408000	6767727456239 3111616000
(6,3,0,1,0,0,0,0)	1257728472000	1732585809034 08000	4119867605858 0040000	1372118443252 292736000
(6,2,2,0,0,0,0,0)	4158222868800	7046220529988 64000	1233046298172 79310400	3909073678785 071596800
(6,2,0,0,0,1,0,0)	0	1321295976000 0	1622701296216 0000	5133078490700 16000
(6,1,1,0,1,0,0,0)	18162144000	2908798129036 800	3900899678339 66400	8878224189875 040000
(6,1,0,2,0,0,0,0)	-29708078400	3782573493098 400	3765735172855 65600	8073918128754 878400
(6,1,0,0,0,0,0,1)	0	0	0	1525620096000
(6,0,2,1,0,0,0,0)	39048609600	3922440099177 600	4216847481742 89600	9488751482136 326400
(6,0,1,0,0,0,1,0)	0	0	1140128589600 0	2992213224000 00
(6,0,0,1,0,1,0,0)	0	2993705114400	2826532308600 00	4377843276206 400
(6,0,0,0,2,0,0,0)	219098880	7017724153440	3736872155016 00	5077606600946 880
(5,4,1,0,0,0,0,0)	-22130572464000	1682338675713 696000	2517029833981 70304000	7789303224014 824512000
(5,3,0,0,1,0,0,0)	0	3294776380896 000	5409544008828 96000	1333465233625 3248000
(5,2,1,1,0,0,0,0)	-1787760374400	6284200262820 4800	5812056128521 843200	1312789211238 01651200
(5,2,0,0,0,0,1,0)	0	0	6211453248000 0	1709420993280 000
(5,1,3,0,0,0,0,0)	-506723817600	3751853311926 7200	3132535130899 574400	7172430859043 0496000
(5,1,1,0,0,1,0,0)	0	4146266124000 0	4244305074609 600	6919307251496 6400
(5,1,0,1,1,0,0,0)	-8454045600	3012440014584 00	1545367613549 7600	2217311142686 64000
(5,0,2,0,1,0,0,0)	2300538240	2077065166176 00	1010251370288 1600	1459565724692 27520
(5,0,1,2,0,0,0,0)	0	1856348846352 00	1016799290226 7200	1544254201473 98400
(5,0,1,0,0,0,0,1)	0	0	10897286400	305124019200
(5,0,0,1,0,0,1,0)	0	13556743200	1339782444000	1559933575200 0
(5,0,0,0,1,1,0,0)	-12972960	328785256800	9569426106240	8715238852320 0
(4,6,0,0,0,0,0,0)	-16064416368000	7546073994459 00000	9053361926315 5068000	2693407049621 824536000
(4,4,0,1,0,0,0,0)	-706810104000	5056829224246 8000	4380607876576 932000	9989831036775 1896000
(4,3,2,0,0,0,0,0)	-1053071379360 0	2091637925657 28000	1516896067322 9563200	3388665899962 06272000
(4,3,0,0,0,1,0,0)	0	3718623308400 0	4691679092100 000	8274594557829 6000
(4,2,1,0,1,0,0,0)	-51156705600	1539702719755 200	8242313859299 5200	1293660907505 366400
(4,2,0,2,0,0,0,0)	-76891815000	1777952957180 400	7515080018425 9800	1143919158414 832800
(4,2,0,0,0,0,0,1)	0	0	68108040000	1907025120000
(4,1,2,1,0,0,0,0)	-211437626400	4428537322809 600	1944446623983 86400	3166176133453 132800
(4,1,1,0,0,0,1,0)	0	189794404800	2216235621600 0	2657938963680 00
(4,1,0,1,0,1,0,0)	-97297200	6081356080800	1796336906364 00	1716967407355 200
(4,1,0,0,2,0,0,0)	-420900480	5784319981440	1431856730304 00	1333215560476 800
(4,0,4,0,0,0,0,0)	-81215053920	8357202395942 40	3122418164065 2000	5251702651023 62880
(4,0,2,0,0,1,0,0)	-90810720	5196693902400	1462580202283 20	1391936343557 760
(4,0,1,1,1,0,0,0)	-1444322880	1727523894096 0	4170902769633 60	3976982859769 920

(Table continued)

Table II. *(Continued)*

(4,0,0,3,0,0,0,0)	0	1105296192000	36844114507200	385567787404800
(4,0,0,1,0,0,0,1)	0	0	1945944000	18162144000
(4,0,0,0,1,0,1,0)	0	2685402720	51178327200	325474269120
(4,0,0,0,0,2,0,0)	3019380	5039612760	66669171660	386439864720
(3,5,1,0,0,0,0,0)	-2930159232000	185772231524880000	1175517964891 6704000	2597905482230 29824000
(3,4,0,0,1,0,0,0)	0	767532205440000	43990547144544000	740549358909360000
(3,3,1,1,0,0,0,0)	248821372800	12864991091752800	501284858001127200	8119100892134232000
(3,3,0,0,0,0,1,0)	0	158162004000	2448484038 0000	30446564148 0000
(3,2,3,0,0,0,0,0)	72547675200	10193084178076800	366725596128091200	6030401621051808000
(3,2,1,0,0,1,0,0)	0	2351417468 4000	749970774352800	7770852611121600
(3,2,0,1,1,0,0,0)	8562153600	89866829052000	2061466327720800	20981444834750400
(3,1,2,0,1,0,0,0)	7042875840	96788164112640	2292542113942080	24832599991879680
(3,1,1,2,0,0,0,0)	1427025600	169724132978400	3673228045687200	41115190582065600
(3,1,1,0,0,0,0,1)	0	0	36324288000	339026688000
(3,1,0,1,0,0,1,0)	0	52540488000	1025512488000	6660058204800
(3,1,0,0,1,1,0,0)	218978760	216354138000	2808478272600	16961127062880
(3,0,3,1,0,0,0,0)	8051883840	108771264201600	2206822363986240	26983778490789120
(3,0,2,0,0,0,1,0)	0	48694726080	943523380800	6136746295680
(3,0,1,1,0,1,0,0)	772251480	510862912560	6198859747080	37332553658400
(3,0,1,0,2,0,0,0)	0	143857153440	1992391521120	12730749071040
(3,0,0,2,1,0,0,0)	0	113539345920	1519496858880	9700522191360
(3,0,0,0,1,0,0,1)	0	5765760	86486400	403603200
(3,0,0,0,0,1,1,0)	513240	98389200	741522600	2910616800
(2,7,0,0,0,0,0,0)	2233943712000	266155141412 16000	1457359948044000000	31974940821567744000
(2,5,0,1,0,0,0,0)	-46162116000	3124344335112000	117387600990660000	1963078820919216000
(2,4,2,0,0,0,0,0)	3369834468000	16245484922066400	527928806340936000	8731401545822155200
(2,4,0,0,0,1,0,0)	0	5862336480000	235381386240000	2801236261824000
(2,3,1,0,1,0,0,0)	15437822400	207767511151200	4711099569976800	54797481418680000
(2,3,0,2,0,0,0,0)	98810712000	165855142252800	3318622886779200	37785861111398400
(2,3,0,0,0,0,0,1)	0	0	45405360000	423783360000
(2,2,2,1,0,0,0,0)	275711436000	749239596705600	14392301780656800	175069440901555200
(2,2,1,0,0,0,1,0)	0	210125916000	4265833572000	28382285131200
(2,2,0,1,0,1,0,0)	183783600	1012258447200	13623456646800	90480068078400
(2,2,0,0,2,0,0,0)	2400297900	1111523092680	12744109958580	84188117373360
(2,1,4,0,0,0,0,0)	168383255040	168737093965440	3033036396149760	37343296560810240
(2,1,2,0,0,1,0,0)	171531360	1333242630720	16876415556000	111941728778880
(2,1,1,1,1,0,0,0)	11913861960	6519407534640	73875801799800	535102774446240
(2,1,0,3,0,0,0,0)	0	1045923278400	13091922043200	103986144662400
(2,1,0,1,0,0,0,1)	0	129729600	1945944000	9081072000
(2,1,0,0,1,0,1,0)	5645640	1948586640	14535120600	57988650720
(2,1,0,0,0,2,0,0)	14785680	1261172640	8806215600	35174193600
(2,0,3,0,1,0,0,0)	0	999382063680	12707386211520	108762949975680
(2,0,2,2,0,0,0,0)	10822151340	5321827110600	55040664418740	467298864592560
(2,0,2,0,0,0,0,1)	0	121080960	1816214400	8475667200
(2,0,1,1,0,0,1,0)	16936920	5747742000	42308065800	169434064800
(2,0,1,0,1,1,0,0)	71351280	5154108960	34528974480	139292112960
(2,0,0,2,0,1,0,0)	59099040	4041797760	26685378720	108004216320
(2,0,0,0,0,1,0,1)	0	262080	1310400	3669120
(2,0,0,0,0,0,2,0)	32460	514920	2062260	5420400

(1,6,1,0,0,0,0,0)	142270128000	3642670231200000	106333547287968000	178066084477881600
(1,5,0,0,1,0,0,0)	0	28148296176000	623630511504000	7603327315584000
(1,4,1,1,0,0,0,0)	225008784000	423497608934400	7715480370998400	95507944852320000
(1,4,0,0,0,0,1,0)	0	36828792000	794593800000	5433508080000
(1,3,3,0,0,0,0,0)	451127476800	522476955000000	8978929312953600	114107850000144000
(1,3,1,0,0,1,0,0)	0	1313123011200	19603612828800	156719526163200
(1,3,0,1,1,0,0,0)	8436027600	4508918013600	47284590553200	359477708990400
(1,2,2,0,1,0,0,0)	10352422080	10078769020320	109486761864480	947311539894720
(1,2,1,2,0,0,0,0)	29704474800	10801322532000	110740462232400	987243798340800
(1,2,1,0,0,0,0,1)	0	605404800	9081072000	42378336000
(1,2,0,1,0,0,1,0)	0	6558552000	48648600000	197621424000
(1,2,0,0,1,1,0,0)	124204080	16942205280	113704390800	497095959360
(1,1,3,1,0,0,0,0)	45223738560	11643286374720	110417480933760	1062262836835200
(1,1,2,0,0,0,1,0)	0	12128276160	89297208000	363606122880
(1,1,1,1,0,1,0,0)	263062800	40941220320	262678656240	1216878062400
(1,1,1,0,2,0,0,0)	537897360	54368233920	337568671440	1595568374400
(1,1,0,2,1,0,0,0)	438197760	39137978880	234931656960	1148977589760
(1,1,0,0,1,0,0,1)	0	5765760	28828800	80720640
(1,1,0,0,0,1,1,0)	917280	14239680	55692000	148337280
(1,0,2,1,1,0,0,0)	668107440	107110523520	623404782000	3423800540160
(1,0,1,3,0,0,0,0)	371891520	71910558720	403882839360	2397080125440
(1,0,1,1,0,0,0,1)	0	17297280	86486400	242161920
(1,0,1,0,1,0,1,0)	4084080	61020960	234234000	627506880
(1,0,0,2,0,0,1,0)	3294720	48597120	185328000	497502720
(1,0,0,0,0,0,1,1)	480	2880	7200	13440
(0,8,0,0,0,0,0,0)	33328795500	183735500949000	4813023119404500	89563150851174000
(0,6,0,1,0,0,0,0)	6684678000	38362231908000	678976996698000	9664787668536000
(0,5,2,0,0,0,0,0)	70882812000	69160939848000	1162695810276000	15203804183568000
(0,5,0,0,0,1,0,0)	0	166990824000	2562375816000	23704624944000
(0,4,1,0,1,0,0,0)	0	458594136000	5692318632000	58125923856000
(0,4,0,2,0,0,0,0)	7063056000	3633095466000	34131565868400	319212473580000
(0,4,0,0,0,0,0,1)	0	126126000	1891890000	8828820000
(0,3,2,1,0,0,0,0)	30421591200	10092198916800	90674957973600	843894132681600
(0,3,0,1,0,1,0,0)	82882800	37109872800	235621386000	1182312331200
(0,2,4,0,0,0,0,0)	20953732800	6683339382720	57536643003840	570187662844800
(0,2,2,0,0,1,0,0)	77357280	59174955840	361900899360	1925900296320
(0,2,1,1,1,0,0,0)	738738000	91740448800	530388658800	3192659870400
(0,2,0,3,0,0,0,0)	637837200	59272012800	333091558800	2177842867200
(0,2,0,1,0,0,0,1)	0	21621600	108108000	302702400
(0,2,0,0,0,2,0,0)	3789240	68978000	254796360	730810080
(0,1,3,0,1,0,0,0)	739939200	118612253760	654906732480	4689721196160
(0,1,2,2,0,0,0,0)	756756000	72305513280	397720683360	3177608353920
(0,1,2,0,0,0,0,1)	0	40360320	201801600	565044480
(0,1,1,0,1,1,0,0)	23783760	477757280	1667505840	5080595520
(0,1,0,2,0,1,0,0)	16576560	350269920	1191350160	3733329600
(0,1,0,0,0,1,0,1)	7280	43680	109200	203840
(0,0,2,0,2,0,0,0)	18018000	687246560	2206027824	7730538816
(0,0,1,2,1,0,0,0)	14414400	957116160	2966483520	11058727680
(0,0,1,0,1,0,0,1)	32032	192192	480480	896896
(0,0,0,4,0,0,0,0)	888030	339073020	1021968090	4041849240
(0,0,0,2,0,0,0,1)	25740	154440	386100	720720
(0,0,0,0,0,0,0,2)	2	4	6	8

One of us (R.Z.R.), discovered this algorithm empirically. The following beautiful proof of its correctness is due to J. Norman Bardsley, to whom the authors are very grateful.

Consider any graph with N vertices and symmetry number S. Assign, in order, the indices 1 to N to the vertices. At each stage, the next vertex can be chosen according to any topological relation that may depend on the indices previously assigned. Each ordering of the vertices λ is given a weight ω_λ. Initially, that weight is set to unity. If at any stage in the assignment of indices the choice of the next vertex is arbitrary with degeneracy d, then d copies of the graph are made corresponding to the assignment. The weight of each copy is reduced by the factor d. When an ordering is complete, the connection matrix is constructed. Two orderings λ and λ' are equivalent if their matrices are identical. Then:

1. There are exactly $1/S$ orderings in each equivalence class.
2. Each ordering in the same class has the same weight.
3. If $\sum'$ represents a restricted sum over orderings including only one member of each class, then

$$\sum_\lambda{}' \omega_\lambda = S \tag{A1}$$

since

$$1 = \sum_\lambda \omega_\lambda = \frac{1}{S} \sum_\lambda{}' \omega_\lambda \tag{A2}$$

ACKNOWLEDGMENTS

The authors would like to thank D. Jasnow for many enlightening discussions. We also thank J. Norman Bardsley, R. Griffiths, and Glenn Whitehead for ideas regarding graph generation. This work was supported in part by the National Science Foundation.

REFERENCES

1. R. Roskies, *Phys. Rev. D* **24**:5305 (1981); D. S. Gaunt, in *Phase Transitions: Cargese 1980*, M. Levy, J. C. LeGuillou, and J. Zinn-Justin, eds. (Plenum Press, New York, 1982); M. Ferer and M. J. Velgakis, *Phys. Rev. B* **27**:2839 (1983); and references cited in these works.
2. C. M. Bender, F. Cooper, G. S. Guralnik, and D. H. Sharp, *Phys. Rev. D* **19**:1865 (1979).
3. J. M. Kincaid, G. A. Baker, Jr., and L. W. Fullerton, Los Alamos Report LA-UR-79, 1575:469 (1979); G. A. Baker and J. M. Kincaid, *J. Stat. Phys.* **24**:469 (1981).
4. R. Roskies, *Phys. Rev. B* **23**:6037 (1981).

5. M. J. Levine, U.S. AEC Report No. CAR-882-25 (1971) [available from the author at Carnegie-Mellon University].
6. A. C. Hearn, REDUCE version 3.0, The Rand Corporation, Santa Monica, California (1983).
7. C. M. Bender, F. Cooper, G. S. Guralnik, R. Z. Roskies, and D. H. Sharp, *Phys. Rev. D* **23**:2976 (1979).
8. T. T. Wu, B. M. McCoy, C. A. Tracy, and E. Barouch, *Phys. Rev. B* **13**:316 (1976).
9. M. F. Sykes, D. S. Gaunt, P. D. Roberts, and J. A. Wyles, *J. Phys. A* **5**:640 (1972).
10. M. A. Moore, D. Jasnow, and M. Wortis, *Phys. Rev. Lett.* **22**:940 (1969).
11. B. G. Nickel, in *Phase Transitions: Cargese 1980*, M. Levy, J. C. LeGuillou, and J. Zinn-Justin, eds. (Plenum Press, New York, 1982).
12. C. J. Colbourn and R. C. Read, *Int. J. Computer Math. A* **7**:167 (1979).